

COMPUTER MODELING OF THE  
ELECTROENCEPHALOGRAM

William Eby Stockslager

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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

COMPUTER MODELING OF THE  
ELECTROENCEPHALOGRAM

by

William Eby Stockslager

June 1974

Thesis Advisor:

G. Marmont

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Computer Modeling of the Electroencephalogram

by

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Lieutenant, United States Navy  
B.S., United States Naval Academy, 1968

Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the  
NAVAL POSTGRADUATE SCHOOL  
June 1974



## ABSTRACT

A computer modeling of the electroencephalogram (EEG) is described based on current research in the field of EEG analysis using the digital computer.

The tegule is defined and possible sources are discussed. The EEG is modeled both in the time and frequency domains.

Modeling revealed that patterns of tegules may be detected in the frequency domain. Sinusoids enclosed in a flat-topped cosine envelope are the most commonly observed tegule shape found in processed EEG data.



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The moral support and encouragement of my wife, Lois, has contributed significantly to this thesis.



## I. INTRODUCTION

### A. BIOENGINEERING

The man-machine interface has been a problem area since man started building machines. In general, a machine is designed around the task to be performed with little or no regard given to the man who will have to operate it. Examples of such designs can be found everywhere in the Navy, the engineering world and the home. As machines have become increasingly complicated and sophisticated the ability of man to adapt to the machine has become more difficult. This has forced him to look for new methods for the operation of such machines.

One approach has been to design special computers to operate the machine. In many cases this approach has proven to be quite successful and sometimes absolutely necessary. But in other cases the computer has only aggravated the problem and proven to be both excessively expensive and often times redundant.

A relatively new approach to the interface problem has been to create an intimate interface between the machine and the man. This approach capitalizes on some of man's greatest attributes, namely his abilities of pattern recognition and discrimination. The solution to the problem of creating this type of interface has emphasized the importance of the field of bioengineering.





One of the objectives of bioengineering has been the determination of the most useful properties that man inherently possesses. To this end, the study of the bioelectrical properties of man's body has become a prime interest. Such a study has been undertaken by the Electrical Engineering Department of the Naval Postgraduate School at Monterey, California. The research project is directed by Professor George Marmont, Professor of Electrical Engineering.

## B. THE RESEARCH PROJECT

Knowledge of the bioelectrical properties of man's brain has in recent years grown rapidly. The advent of microelectronics and the digital computer has provided powerful tools for the analysis of the electroencephalograph (EEG). A sophisticated system that makes effective use of these tools has been assembled at the Naval Postgraduate School.

One of the main goals of the bioengineering research at the Postgraduate School is to develop a means of dissecting the EEG patterns of a healthy man in a manner that results in one being able to effectively use these patterns in any one of several ways. It is postulated that by observing certain specific EEG patterns one can determine the mental process that is being performed by an individual. Such mental states as deep sleep, light sleep, awake but relaxed, and problem solving have already been determined through the analysis of EEG patterns.



It is proposed to feedback to a person an indication of how well he is using his brain in solving a problem or learning to perform a task based on his EEG patterns. It is believed that such a feedback scheme would enhance the individual's learning ability by decreasing the time necessary to learn the material or task as well as increase his ability to retain what he has learned. This could well reduce the time required to train a pilot to fly a new aircraft, etc.

Another use is proposed to feedback to an individual when his brain has detected a certain pattern such as is the objective in observing ASW grams. This could markedly reduce the reaction time inherent in such tasks.

Other techniques for the use of appropriately detected brain wave patterns are numerous and of great potential value to the Navy. NAV-ELEX has recognized this potential and has elected to fund the research project at the Naval Postgraduate School for the past two years.

### C. THE AUTHOR'S CONTRIBUTIONS

In addition to frequent participation as a subject in the research project, the author has contributed in the collection, recording, and analysis of EEG data. The author's prime contribution to the research project and the subject of this thesis has been in modeling the EEG using digital computer techniques.



## II. BACKGROUND

### A. THE ELECTROENCEPHALOGRAM

Electrical activity originating from the cerebral cortex was first observed by the English physician Caton [Ref. 1] in his laboratory in 1875. The next significant step in the study of the electrical waves of the brain was not until 1929 when Hans Berger published his studies of the brain waves of man as recorded through electrodes on the scalp [Ref. 1]. Since then the primary study and application of the electroencephalogram has been the detection and classification of anomalies in the human brain.

Today an increasing emphasis has been placed on applying EEG technology to new applications involving healthy man. The device commonly seen in advertisements today that indicates when a person has achieved a mental state characterized by alpha EEG is a simple bio-feedback loop. The device detects the presence of alpha waves and responds with an audio signal. The person wearing the device attempts to maintain alpha as indicated by the audio signal of the device by relaxing to a state of minimum thought.

### B. EEG WAVE FORM CLASSIFICATION

A crude classification of the EEG waveforms into frequency bands given certain Greek alphabetic symbols has become the accepted standard of classification in EEG technology (see Figure 2-1). This



particular classification is most suited to general classifications of the state of wakefulness and attention. For example, the prevalent waveform during light sleep is the theta, during a very relaxed but awake state is the alpha, during awake and alert state is the beta [Ref. 1].





FREQUENCY BAND (Hz)	CLASSIFICATION SYMBOL
0 - 3.5	$\delta$
4 - 7	$\theta$
8 - 13	$\alpha$
14 - 50	$\beta$

Figure 2-1. Classification of the EEG wave forms [Ref. 2].



### III. METHODS

This section describes the laboratory setup used in obtaining the EEG data which was used as a basis for modeling the EEG. Figure 3-1 is a block diagram of the experimental setup. Salient features of the setup are described in the following paragraphs.

#### A. THE DIGITAL COMPUTER

The increased speed of modern specialized digital computers has brought into reality the long desired ability to process information on a real time basis with a high degree of accuracy. This technique is the crux of the present EEG research project.

A DEC PDP 11/40 digital processor with Time Data microprogrammed fast Fourier Transform processor is the heart of the data collection and processing system used in the research program. This versatile machine permits real time analysis of eight channels of EEG information simultaneously. It can perform various mathematical functions on the data including spectrumanalysis, cross-correlation, autocorrelation, averaging and digital filtering.

##### 1. Data Processing

The analog EEG signal is converted into a digital format through the use of an analog-to-digital converter that samples the incoming data for each channel at a predetermined rate. The digitalized data is placed



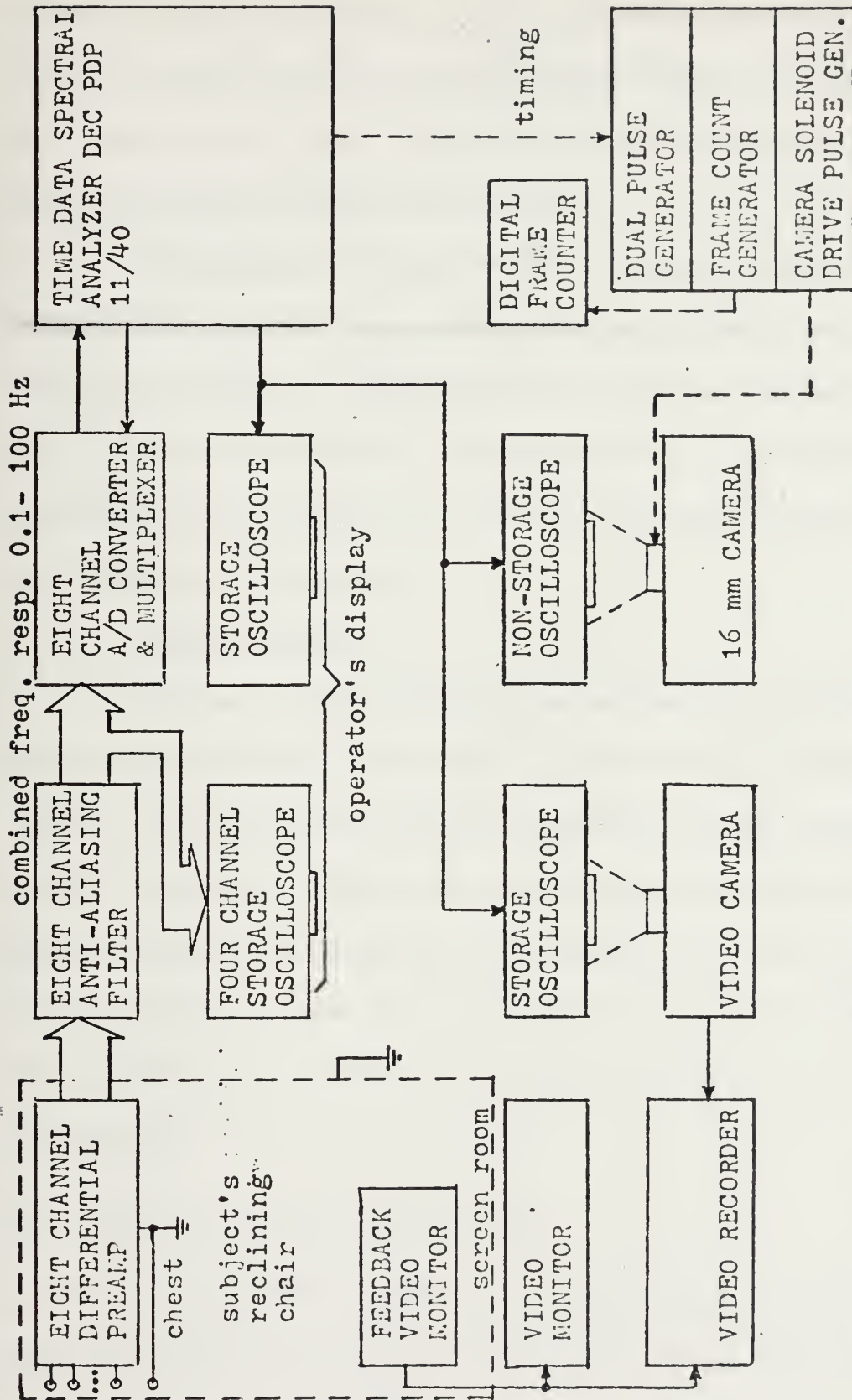


Figure 3-1. Block diagram of experimental setup.



into one of two buffers. When a buffer is filled, its contents are automatically made available for processing by the digital processor. The digital processor removes the data from this buffer and clears the buffer for another epoch of data. In the meantime, the second buffer is being filled with the next epoch of data and so on.

The digital processor processes the data obtained from the buffer in the particular manner that it is programmed to do and displays the processed data on a small local CRT as well as a larger remote CRT. All this occurs within one second or less. Consequently, the processed data is available for analysis less than one second after the raw EEG signal was obtained.

## 2. Digital Filtering

The heart of digital filtering is the Time Data's hardware fast Fourier transform unit. This device enables the digital processor to convert data that is in the time domain into the frequency domain within a few milliseconds. While in the frequency domain, filtering can be performed by simply zeroing the frequencies that are not desired. Once this has been done, the same module is used to transform the filtered data back from the frequency domain for further processing and display.

## B. ANALOG SIGNAL CONDITIONING

The detected EEG signal must be amplified and filtered while in the analog state prior to its being input to the analog-to-digital





conversion unit. Due also to the large amount of activity within the brain that generates the EEG, the placement and reference for the electrodes are important in dissecting the EEG while it was still in its analog state.

### 1. Interference Reduction

Since the raw EEG signal as detected by scalp electrodes is usually in the range of a few to about 100 microvolts, it is highly susceptible to interference. Consequently, great care must be taken to reduce interference and to provide an environment that is as free as possible of electromagnetic interference. Such an environment was provided by the use of a screen room.

The second step is to combat the ever present 60 Hz that permeates even screen rooms. To tackle this problem, a differential amplifier with a high common mode rejection ration (CMRR) was used. The differential amplifier was of two stage design with each stage having a differential input. This produced a differential amplifier with an CMRR of 24000:1 [Ref. 6].

In addition to the differential amplifiers, the electrode leads were kept as short as possible to reduce the common mode pick up of 60 Hz. This effectively eliminated any problems with outside interference.

### 2. Electrode References

There were three basic electrode references used in observing the EEG. They were the bipolar, unipolar and averaging reference



which are described in the following paragraphs.

a. Bipolar

The bipolar reference is illustrated in Figure 3-2a. This method references one electrode on the scalp to another electrode on the scalp. This method is particularly useful in limiting pick up to the area of the cortex that is between the electrodes.

b. Unipolar

The unipolar electrode reference is illustrated in Figure 3-2b. This method uses a single electrode as the reference electrode for all other electrodes. In the research project, the unipolar reference was most commonly used with the reference electrode placed on the left ear lobe. Such an arrangement of the electrodes permits detection of the EEG directly beneath the electrode. The reference electrode on the ear lobe can be considered to be effectively referencing the center of the brain to all other electrodes. Hence, any source of the EEG that is near the center of the brain would be detected nearly equally by all electrodes. This is sometimes undesirable. This led to the development of the averaging method.

c. Averaging

The latest electrode referencing technique used in the research project was the averaging method. Figure 3-3 is a schematic diagram of the averaging circuit used to reduce the average activity as detected by all electrodes. This method is useful in dissecting the EEG



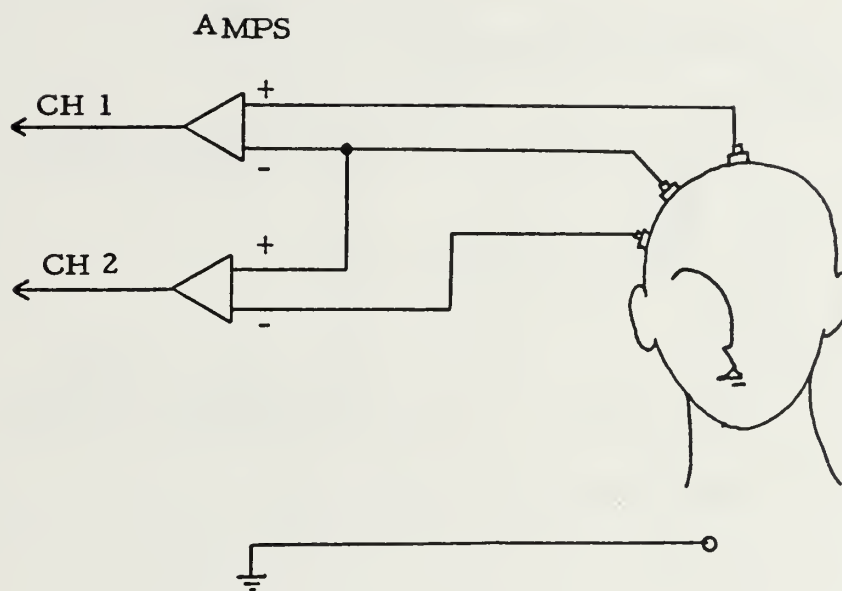


Figure 3-2a. The bipolar electrode referencing technique using only two amplifiers.



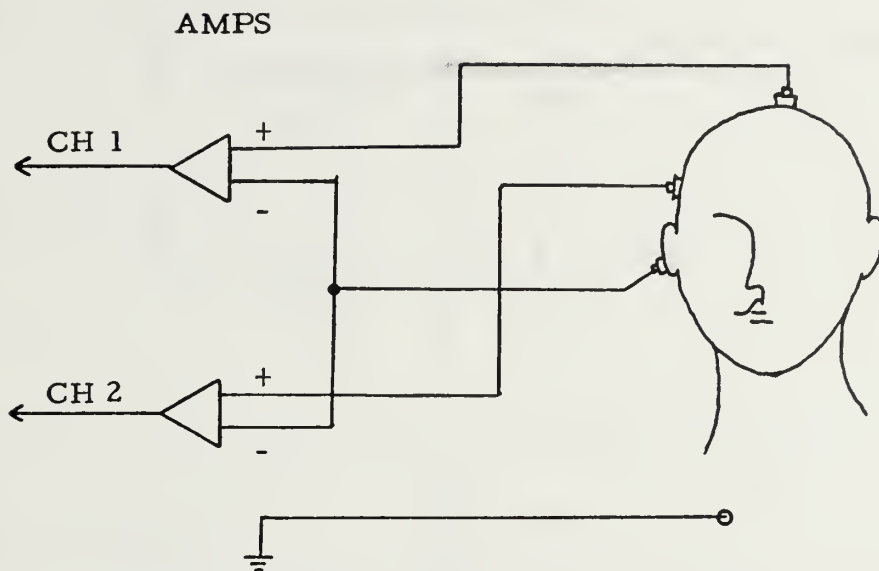


Figure 3-2b. The unipolar electrode referencing technique using only two amplifiers.





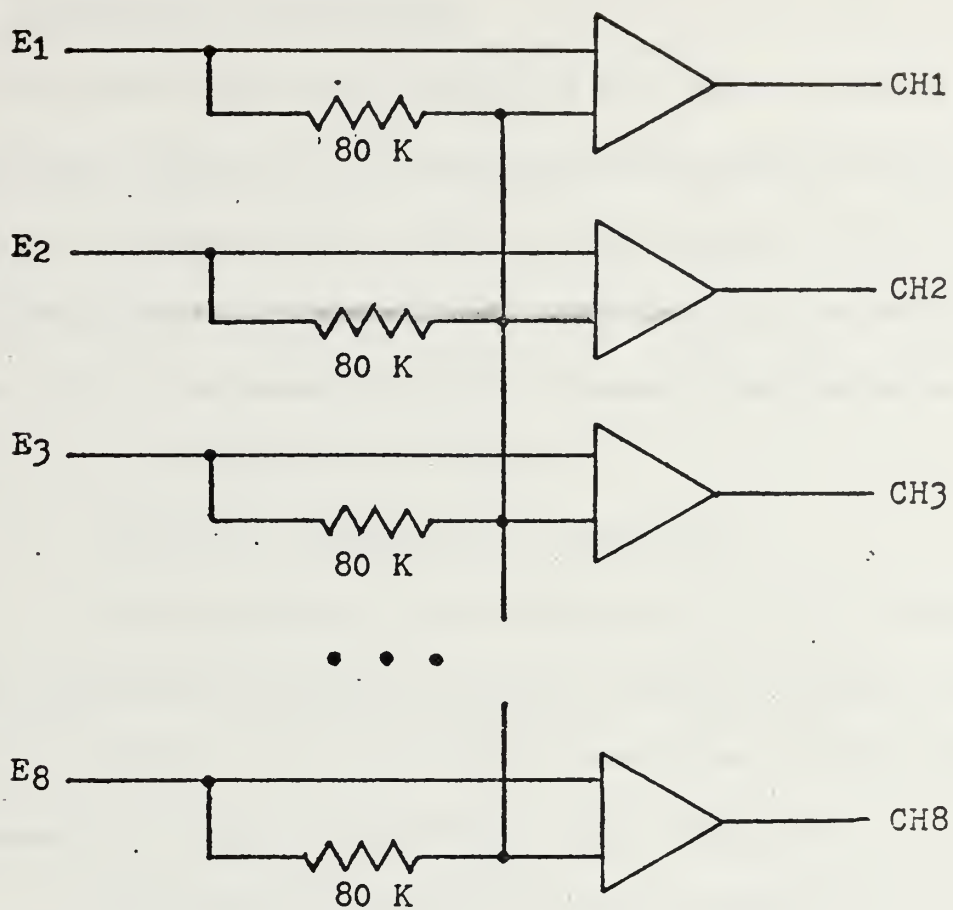


Figure 3-3. Averaging circuit used for averaging electrode references.



that is generated in the cortex under each electrode while ignoring that part of the EEG which is common to all electrodes.

### 3. Amplification and Filtering

The EEG signal must be amplified from the microvolt range to the one volt range prior to its being input to the analog-to-digital unit. It should also be filtered with a high frequency limit that is only slightly higher in frequency than the maximum frequency that is desired to be observed. This is necessary for proper frequency discrimination and is known as an anti-aliasing filter [Ref. 6].

#### a. Operational Amplifiers and Components

Amplification of a 50 microvolt signal, as is commonly the signal amplitude of the EEG, requires a very quiet amplifier in a differential amplifier circuit. Operational amplifiers of high quality (and expense) are capable of providing the quiet amplification necessary.

The operational amplifiers used were found to be susceptible to vibration even though they were advertised as being integrated circuits. Destructive examination of one of the op-amps revealed that it was constructed of several integrated circuits connected together by very fine gold wires! Such a construction would obviously be susceptible to vibrational problems. It is recommended that operational amplifiers of monolithic construction be used for EEG amplification work as soon as they become available.



## b. Filtering

A Fourth-order Butterworth filter was used in filtering the EEG within the bandwidth of 0.1 to 100 Hertz. This filter provided a very linear response and the anti-aliasing limits desired.

## C. COMPUTER PROGRAMS

The EEG has been observed and used in its amplified analog form for medical as well as research purposes until recent years. Now that real time computer analysis is available, a new dimension in the study of the EEG has been added to these fields. A major part of the research has been in the area of digital processing of the EEG signal. Computer programs have been written to process the EEG and present it in the frequency domain as well as the time domain. Cross correlation in the time domain and cross spectra in the frequency domain are just two of many techniques available using digital computer. These techniques provide powerful tools in the real time dissection of the EEG.

Nearly all of the computer programs used in the research project that are involved with displaying the processed EEG in either the time or frequency domain use digital filtering. The technique used is to transform the sampled EEG into the frequency domain, then zero those frequencies that are desired to be filtered out. The remaining frequencies are then transformed back into the time domain and the resulting information is displayed and recorded on film. This method provides an ideal filter and enables one to focus in on specific frequency bands that are of interest.



## 1. Effects of Digital Filtering

One of the questions that one in research must continually ask is whether or not the data he is recording is accurate and to what degree that the measuring technique used effects the outcome of the measurement. The unique method of digital filtering described in the above paragraph is one such method that requires an answer to the question of what effect it has on the measured data.

When the bandwidth is limited to 10 Hertz, a great deal of information contained in the original EEG signal is lost and the resulting filtered signal is of quite a different envelope shape. In the case where the original frequency falls near the filter's limits, it is shifted in frequency towards the center of the filter's frequency band. A bandwidth of 20 Hertz has similar effects on the EEG signal but distortion is to a much lesser degree.

Once these effects are recognized and taken into consideration during analysis of the processed EEG, one can make effective use of digital filtering to examine the amount and characteristics of EEG activity at particular frequencies. This is a powerful tool in the research effort.

## 2. Definition of Tegule

When the EEG signal is digitally filtered and displayed in the time domain, the most obvious characteristic is that the EEG is composed of many spindle shaped sinusoids. The frequency range of these





sinusoids is from slightly above 10 Hz to frequencies in excess of 80 Hz and may FM slightly.

Rigorous research has revealed that although the frequencies present in the EEG may be harmonically related, they are seldom time coincident. This has been found to be true even of sinusoids of the same frequency detected with electrodes closely spaced together, particularly in the higher frequencies [Ref. 6].

The duration of the sinusoids may vary from as small as 25ms to larger than 200 ms with an envelope that appears either flat-topped cosine or cosine in shape. This, together with the characteristics detailed in the above paragraphs, defines the tegule.

Research has also established that the tegule is not the result of an impulse response from a filter. Figure 3-4 is an example of EEG data which clearly shows tegules with the characteristics as described above.



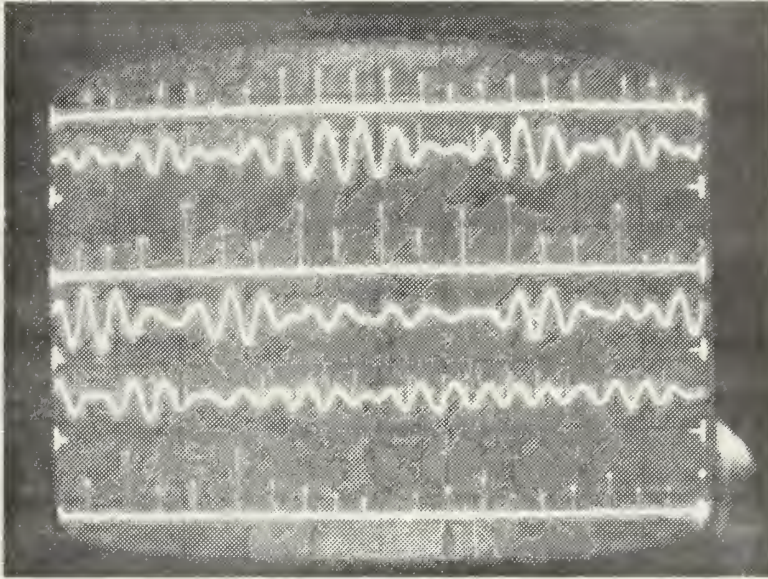


Figure 3-4. Digitally filtered EEG data displayed in the time domain.

Each trace represents 250 ms of successive data. The filter frequencies used were 65 to 105 Hz. (Note: The vertical bars are located at the zero crossings. The height of the bar is proportional to the period of the wave form.)



#### IV. MODELING THE EEG

The physiological structure of the brain as well as the electrical properties of neurons are both very important elements that must be considered in any model involving the observed brain waves in an EEG. The following paragraphs describe these elements as they are considered in computer modeling of the EEG.

##### A. THE CEREBRAL CORTEX STRUCTURE

The brain has three major parts consisting of the cerebrum, the brain stem, and the cerebellum [Ref. 1]. The cerebrum is divided into a left and right hemisphere. The outer part of the cerebrum is a thin layer of densely packed nerve cells and is gray in color and known as the cerebral cortex. White in color, the inner portion of the cerebrum consists of many nerve axons. The inner portion of the cerebrum is primarily a large complex of connections between different parts of the cerebral cortex and other parts of the brain. The cerebral cortex is described in further detail in the following paragraphs.

##### 1. Cellular Composition

As described above, the cerebral cortex consists of a large number of neural cells (around 10 billion) that have been classified into six cytoarchitectural layers [Ref. 1]. Within these layers, there are five major cell types found. The most distinguishable cells are the pyramidal and stellate cells (see Figure 4-1).



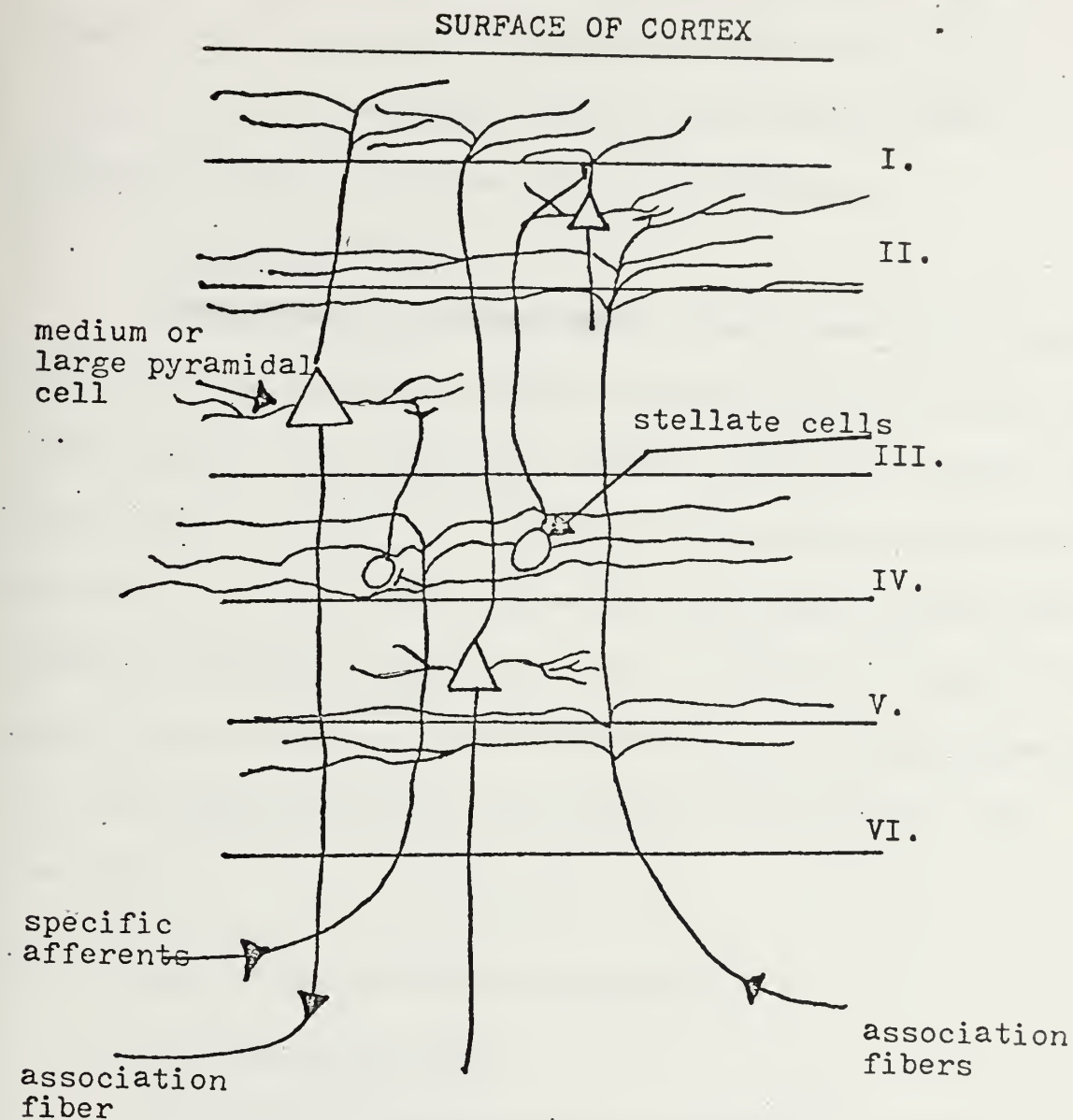


Figure 4-1. Diagram of the six layers of the cerebral cortex.





It is postulated that the stellate cells act mainly as interneurons between specific afferents or association fibers and the pyramidal cells [Ref. 1]. An example of a possible network of these cells is shown in Figure 4-2. It is within the cerebral cortex that the process of thinking and awareness are believed to be primarily located.

## 2. Surface of Cortex

The surface layer of the cortex, layer I, contains relatively few cells but is comprised mostly of a dense tangential fiber plexus of apical dendrites, ascending axons, and axon collaterals in which a very large number of synapses occur [Ref. 2]. The excitation of the apical dendrites near the cell body of pyramidal cells causes a current flow within the cortex (see Figure 4-3). This current flow through the resistance of the extracellular fluid of the cortex causes a voltage to develop. It is this voltage that makes up the EEG and is detected by a scalp electrode.

## B. THE RETICULAR ACTIVATING SYSTEM

### 1. Physiological Location

The reticular activating system (RAS) has been defined to be located within the center of the midbrain (see Figure 4-4). This portion of the midbrain has many functions and is involved in some manner with nearly every sensory input to the brain and all the motor outputs of the brain [Ref. 1, 2 and 3].



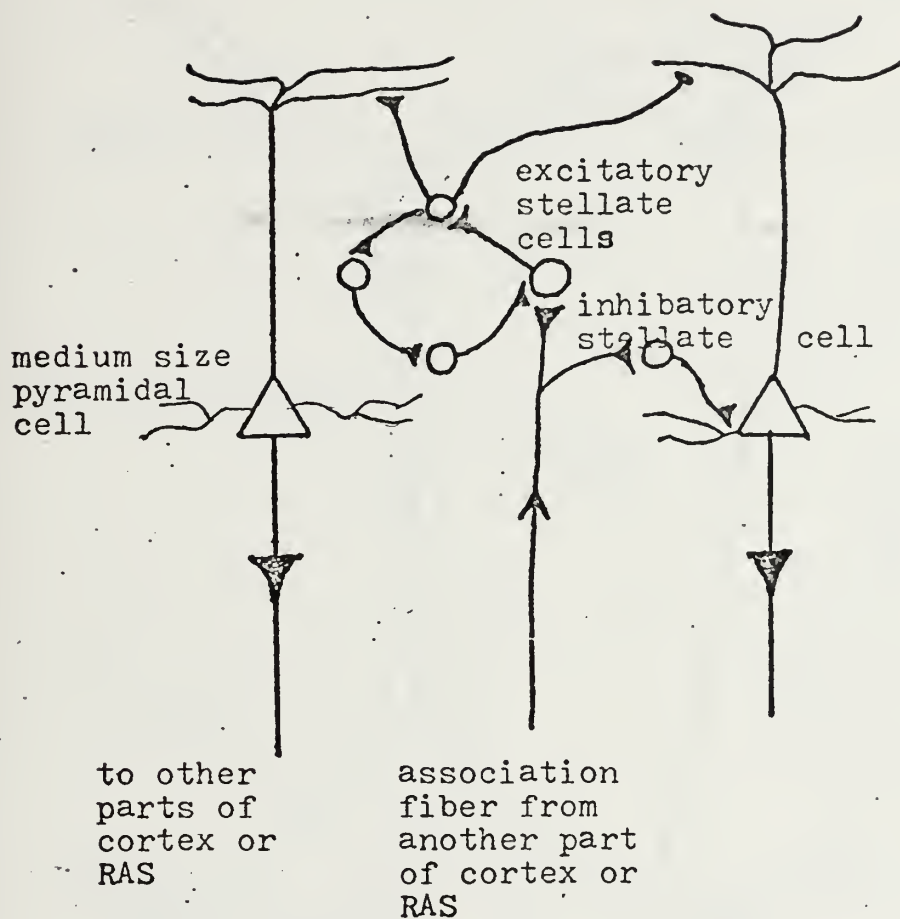


Figure 4-2. Diagram showing a hypothetical circuit involving pyramidal and stellate cells in the cerebral cortex.



# Incoming Action Potential

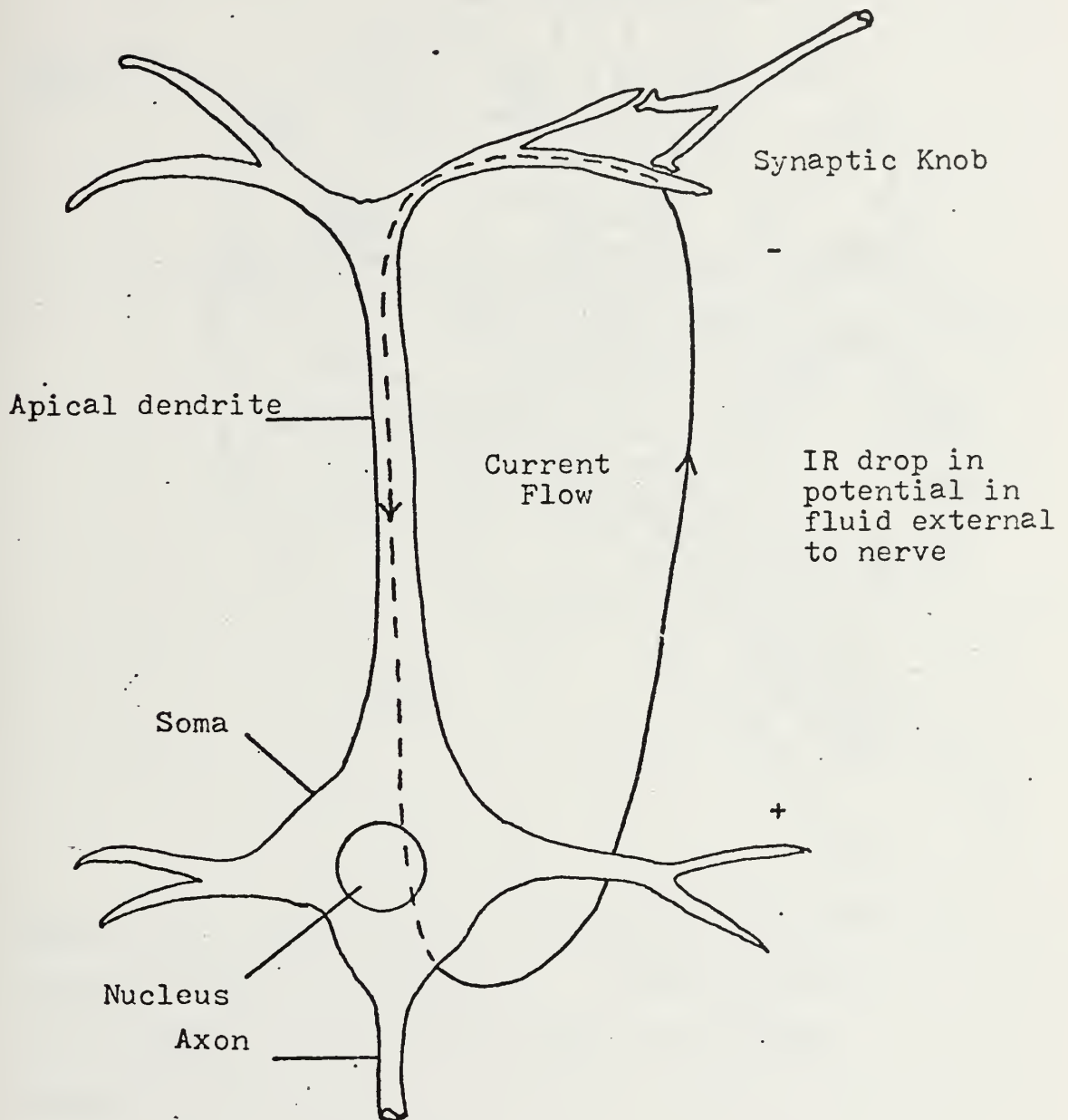


Figure 4-3. A simplified diagram of a pyramidal cell in the cortex.



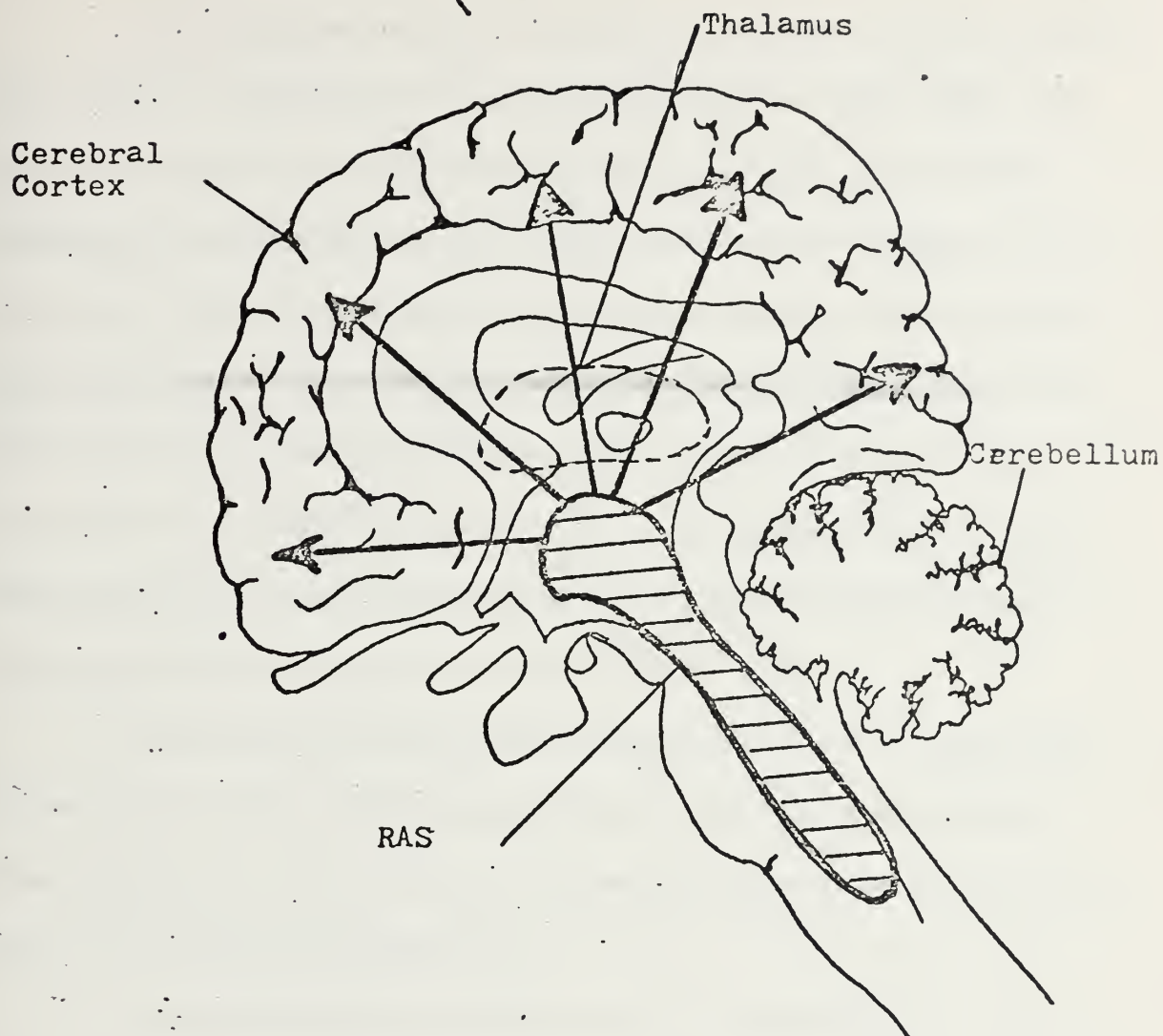


Figure 4-4. Location of the ascending reticular activating system p. 1559 Ref. 3 (hashed region). Arrows indicate the RAS activation of cerebral cortex.





## 2. RAS Activation of Cortex

One of the most interesting functions that the RAS performs is the control of wakefulness and sleep. When one is in light sleep, the EEG is predominately composed of very low frequency wave forms, sometimes termed theta waves. Theta waves seem to originate only in the cortex, since a brain with a lesion separating the cortex from the RAS still possesses the theta waves characteristic of light sleep. One who is awake but resting will have an EEG composed mostly of the commonly known alpha waves. It has been noted that a brain in which the communication pathways between the cortex and RAS have been cut cannot generate the characteristic alpha waves [Ref. 2].

It has also been noted that stimulation of the RAS region will immediately awaken a sleeping animal to a high degree of alertness. Consequently, the role of the RAS in controlling the state of wakefulness has been well established [Ref. 3].

During deep sleep the EEG pattern returns to one very similar to that during an awake state. This phenomena has been termed paradoxical sleep, [Refs. 4 and 5] and is highly correlated with the state of dreaming. The control of the amount of paradoxical sleep is also apparently located within the RAS.

## C. POSSIBLE TEGULE SOURCES

The way in which the switchability of neurons varies has been studied in great detail [Ref. 2]. Taking this phenomena into consideration



one is led to the conclusion that since the brain contains many millions of neurons, that the operation of a neuro circuit is likely to have a probabilistic description. Consequently, the observed tegules most likely result from a large number of neurons triggering, not at the exact same instant, but with some probabilistic distribution about a point in time.

### 1. Alpha Synchronization

A long standing theory is that the alpha wave is evidence of a synchronizing gate [Ref. 2]. If this is the case, then since each cycle of the roughly 10 Hz alpha signal lasts approximately 100 ms, one would expect the neurons of the brain to have the greatest probability of triggering every 100 ms.

The alpha wave form is not exactly sinusoidal but is somewhat scalloped as shown in Figure 4-5. There are roughly 15 ms during which the alpha wave would most likely enhance the probability that a neuron would trigger. Section V of this paper treats this theory in more detail with some modeling examples.

### 2. Recruiting Response

It has been postulated [Ref. 2] that the alpha wave is caused by the combination of reverberating circuits in the thalamus in conjunction with the recruiting response in the cerebral cortex due to stimulation originating from the thalamic region of the RAS. The recruiting response is a delayed, slow rise in potential in the cerebral cortex that becomes increasingly more intense when stimulated at about 10 Hz.



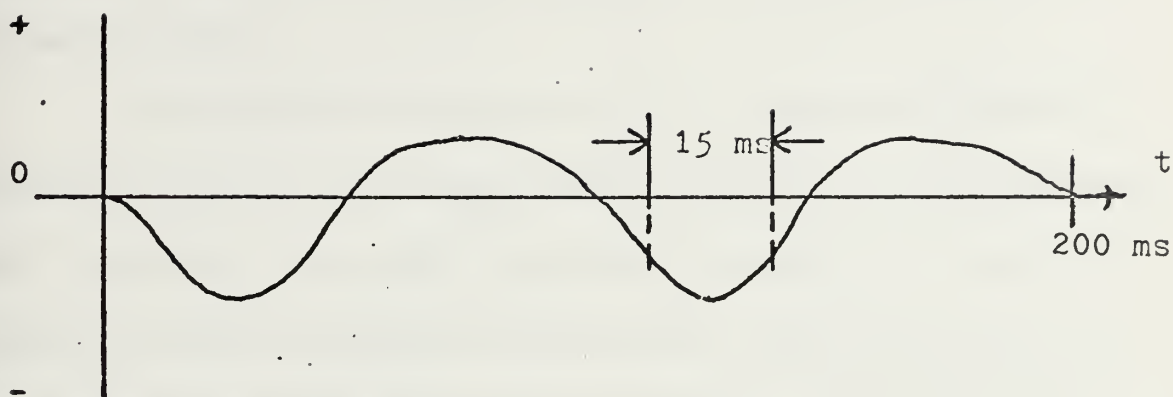


Figure 4-5. Scallop shape of alpha wave form. The 15 ms region during which strobing most likely occurs is indicated.



The reverberating circuits within the thalamus oscillate at between 8 and 13 Hz when the cerebral cortex is strongly stimulated [Ref. 3]. The positive feedback of these two systems could be the source of the alpha wave.

### 3. Reverberating Circuits

An example of a simple reverberating circuit is shown in Figure 4-6. Such a circuit may reverberate until either inhibited or stopped by fatigue.

The process of facilitation may cause additional neurons to become part of the reverberating circuit and thereby increase the circuit path, resulting in a decrease of the output frequency. In a like manner, fatigue or selective inhibition may cause less cells to take part in the reverberation and increase the output frequency. Facilitation and inhibition may be the primary means with which the frequency of tegules is controlled.

Reverberating circuits that expand or contract in size are possible as well as the fixed circuit size shown in Figure 4-3. Such a circuit would explain observed tegules that vary in frequency.

### 4. Frequency Beating

Two or more tegules of relatively long duration that are of nearly equal frequencies may be linearly summed by the detecting electrode to produce a series of short tegules. Such addition of two sinusoids is sometimes referred to as frequency beating. The resulting





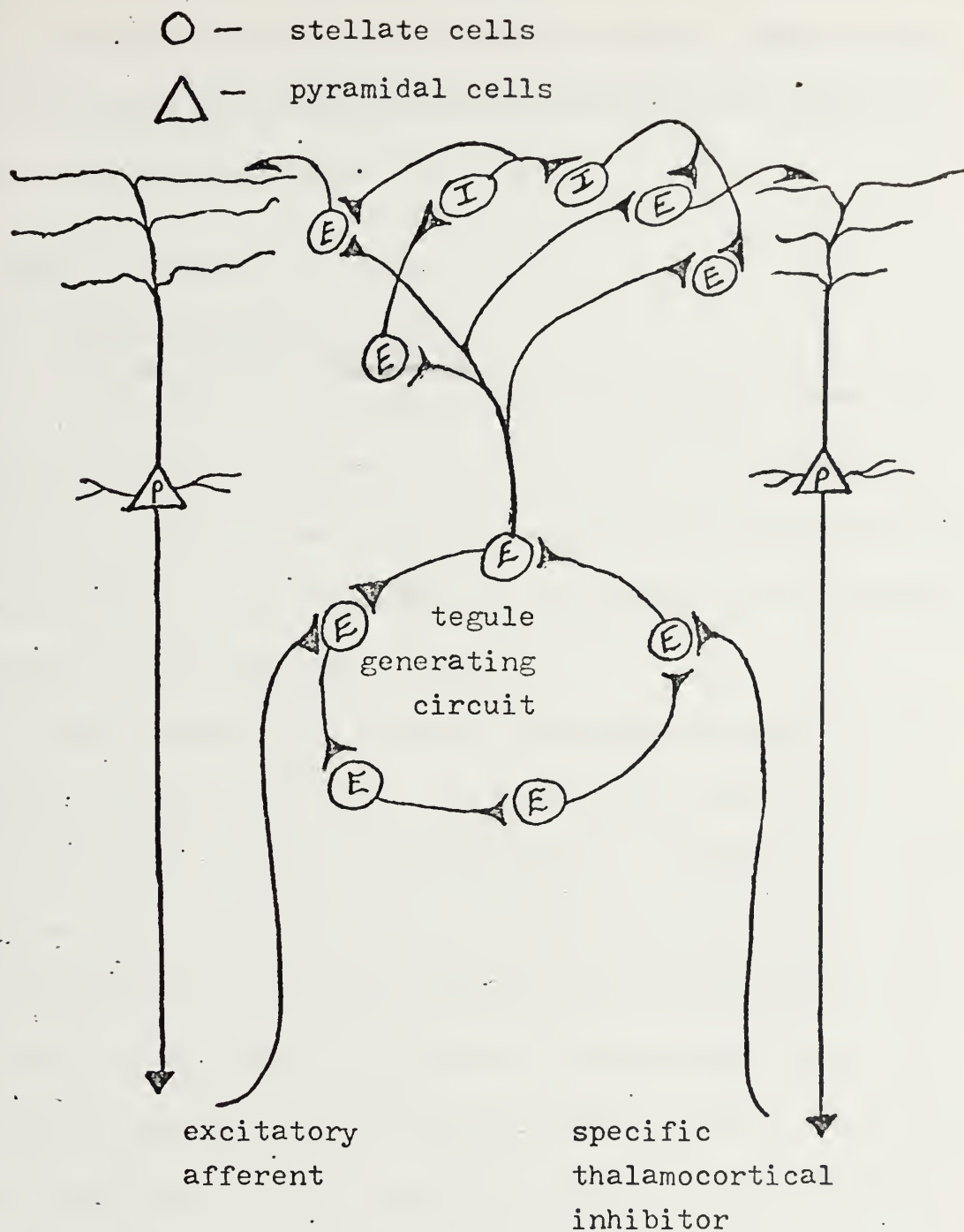


Figure 4-6. Neural circuit diagram for a constant frequency tegule. E- Excitatory, I- Inhibitory, P- Medium Sized Pyramidal Cell.



short tegules have a frequency that is the average of the frequencies of the two sinusoids that were summed to produce them. Their envelope is a cosine shape with a frequency equal to the difference of the frequencies of the original tegules.

#### D. TEGULE CHARACTERISTICS

##### 1. Constant Frequency Tegule

A commonly observed tegule is one of relatively constant frequency for a period of several tenths of a second. The frequency of such tegules range from below alpha to as high as 100 Hertz. Figure 4-6 shows a possible circuit arrangement of neurons that would be capable of generating a constant frequency tegule.

The envelope shape of tegules varies from a general cosine shape to a flat-top cosine shape. The shape of the tegule may be a result of the number of apical dendrites that are stimulated at one time. Inhibition by the stellate cells due to feedback inhibitors within the cortex or inhibitors from the RAS could well be the determining factor in the duration of the tegule and the number of apical dendrites that are stimulated. Another possible explanation for the envelope shape of the tegule might be the strobing of many such circuits at different times and different numbers. The alpha wave form has been suspected as being a strobing signal for some time [Ref. 2] and may well have this effect. If so, one would expect the sinusoids generated to be nearly in



phase with start times within about 15 ms as described in the section on alpha synchronization.

## 2. Linear Period Sweep Tegule

A commonly observed tegule is one which changes frequency as a linear function of the period of the waveform. Such tegules either increase or decrease in frequency or some combination of both. A constant frequency sometimes exists between frequency shifts in a single tegule. The resulting wave form often possesses quite a complicated combination of frequency and period characteristics and are observed most often during a problem solving mental state.

One possible explanation of the source of linear period swept tegules is found in considering path length changes in the size of an oscillating neural circuit. The circuit may consist of a matrix of stellate cells of approximately equal axon length. As oscillation is induced in the circuit, the number of cells in the closed oscillating circuit may vary discretely in a linear fashion, due to either facilitation or inhibition, thus causing the period of oscillation to vary. Hence, the frequency is inversely proportional to the number of cells involved in each ring.

## 3. Nonlinear Frequency Sweep Tegule

Another tegule commonly observed is one which changes frequency in a nonlinear function. The possible explanation for such tegules is quite similar to that of the linear period sweep tegule



described above with the exception that the cell may have different axon lengths and may be added by facilitation or deleted by inhibition from the oscillating circuit as some nonlinear function of the number of cells involved.

## E. COMPUTER MODELING PROGRAM

The computer modeling program used can be divided into major functional parts: sinusoidal type generation, sinusoidal envelope shaping, sinusoidal summation with display, and resultant tegule display. (see Figure 4-7) The modeling program uses the DEC PDP 11/40 computer and the same basic software and hardware as used in the research project. Consequently, the method of filtering and displaying the resultant tegule is identical to the process used in digitally filtering the EEG in the research project. This enables one to examine what effects the digital processing has on the EEG data, since the modeled sinusoid is of known properties.

### 1. Generation of Sinusoids

The computer program uses mathematical functions to generate the appropriate sinusoids. The Time Series Language used in the PDP 11/40 is structured to enable one to manipulate blocks of data. The block sizes are determined by the programmer to suit the sampling rate and epoch size of incoming data. In the case of the modeling program, a block size of 1024 words was chosen to represent one second of simulated time data.





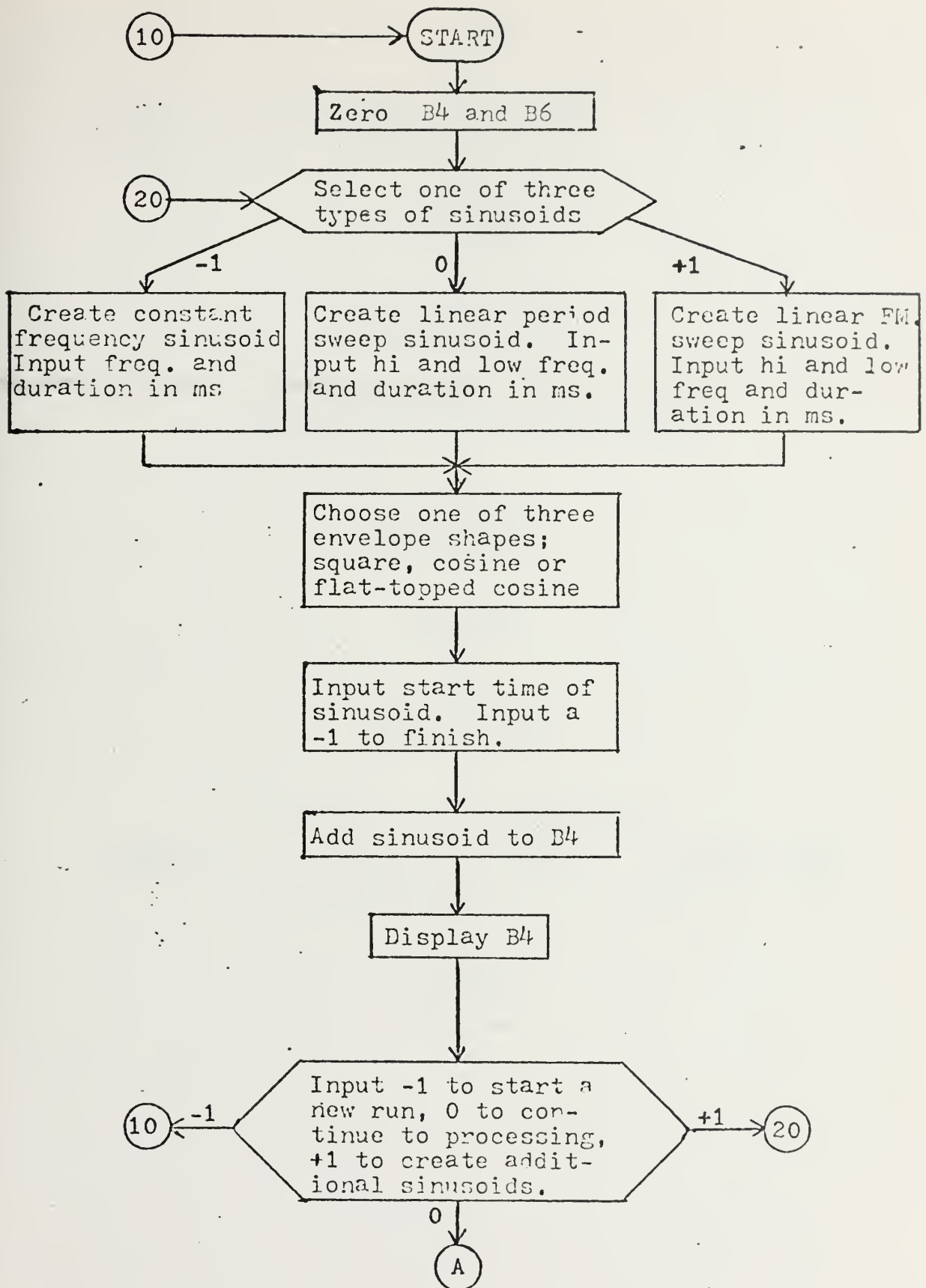
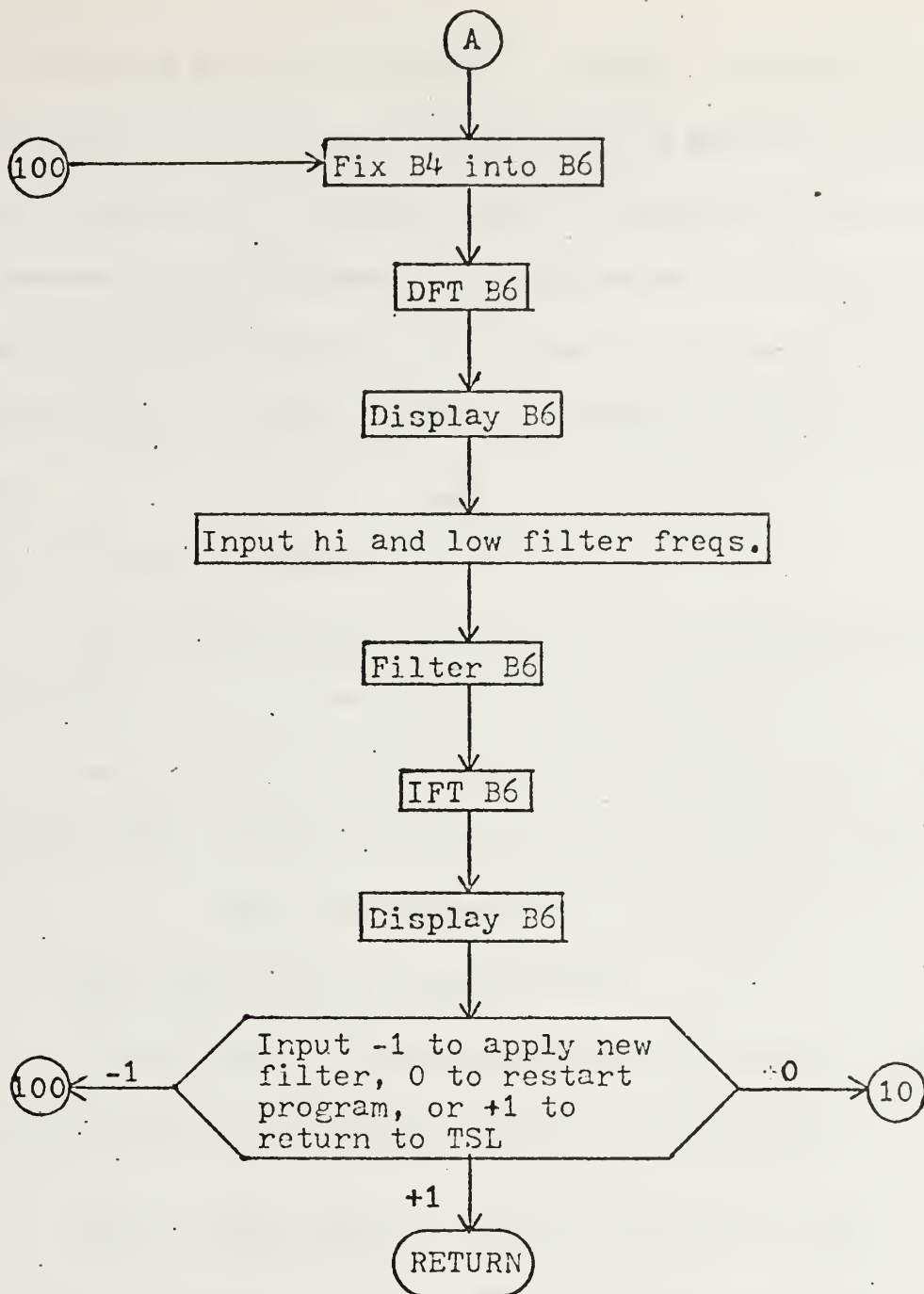


Figure 4-7. Flow chart of computer modeling program.







Each word in a block represents a sample. In this case, there are 1024 samples in one second. Consequently, to generate a sinusoid in the block using discrete samples, one first must load the block with a set of numbers that correspond to the appropriate time function for each sample point, then the sine can be taken for each point, leaving the block with a set of numbers that when displayed, form the desired sinusoid.

a. Constant Frequency Sinusoid

The constant frequency sinusoid was generated using the following formula, where  $f$  represents the desired frequency,  $a$  is  $1/1024$ ,  $n$ , the only variable, is the number of the sample point (varies between 0 and 1023), and  $B(n)$  is magnitude of the  $n$ th block element:

$$B(n) = \sin 2 \pi fna$$

b. The Linear Period Sweep Sinusoid

To generate a sinusoid whose frequency varies as a linear function of the period, the following formula was implimented:

$$B(n) = \sin \left\{ \frac{2\pi \tau}{1/f_2 - 1/f_1} \ln \left[ \frac{(1/f_2 - 1/f_1)f_1 n}{\tau} + 1 \right] \right\}$$

where  $f_1$  and  $f_2$  stand for the start frequency and final frequency, respectively,  $\tau$  is the duration of the sinusoid in seconds and all other terms are the same as defined above.

c. The Linear FM Sweep Sinusoid

Generation of a sinusoid with a frequency that varies between two frequencies as a linear function of frequency was made using the



following formula:

$$B(n) = \sin \left\{ \frac{2\pi n}{1024} \left[ f_1 + \frac{n(f_2 - f_1)}{1024\tau} \right] \right\}$$

where the terms are the same as those defined above.

## 2. Envelope Shape

The very important characteristic of the finite tegule is the shape of its envelope. The computer program enables the programmer to choose one of three envelope shapes; square, cosine, or flat-topped cosine.

## 3. Summation of Sinusoids

The generated sinusoid may be located within the one second block at any location. If and/or other sinusoids may be added as many times and at as many locations as desired. After each summation, the resultant sinusoid is displayed for observation and plotting.

## 4. Filtering of the Sinusoid

When the desired sinusoid has been created, it is ready to be filtered. The FFT is taken of the sinusoid and the magnitude of the FFT is displayed. The filters limiting frequencies are chosen at this point and applied to the FFT, with the results subsequently displayed.

Upon completion of filtering the IFT is taken and the resulting tegule is displayed and may be plotted. At this point, one may choose to use different filter limits on the original sinusoid and proceed through the filtering routine again.





## 5. Record of Events

A record of the procedure used to create the sinusoid and the filter limits applied to give the resulting tegule is made in hard copy form as the program proceeds.

Detailed examples of the output of the modeling program are given in section V.

### F. EEG FREQUENCY SPECTRUM CHARACTERISTICS

The EEG is made up of many tegules during the awake state. During any single second of data, one is likely to find three or more tegules of different frequencies and durations. The frequency spectrum of a series of tegules is unique in that certain frequencies disappear that are otherwise present. Figure 4-8 is an example of the frequency spectrums of six channels of EEG. Note that there are certain discrete frequencies that stand out in every channel while nearby frequencies are low or absent.

#### 1. A Single Tegule

A sinusoid of finite duration has a unique frequency spectrum that is determined by the envelope shape and duration of the sinusoid. A characteristic sinusoid with a cosine shaped envelope is shown in Figure 4-9 with its mathematical representation.

The Fourier transform of the envelope is found by evaluating the intergral as below.



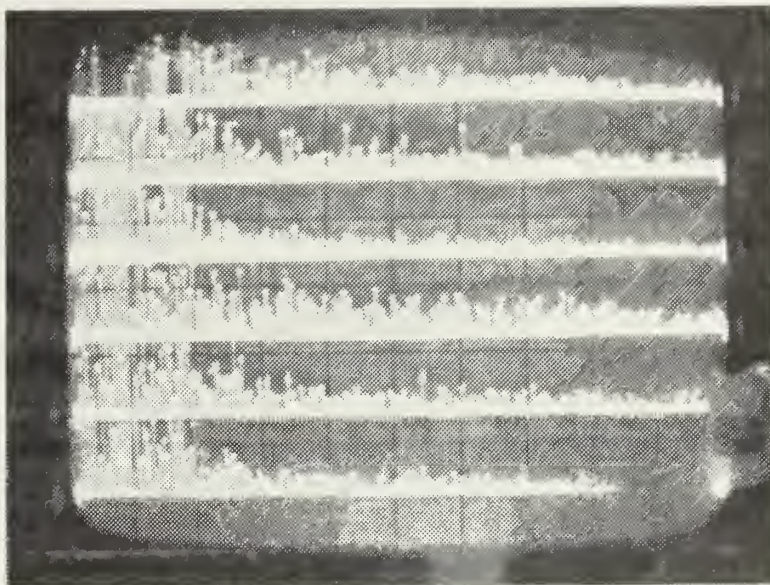


Figure 4-8. Six channels of EEG data displayed in the frequency domain. The X axis represents a range of 0 to 128 Hz. Note the apparent discreteness of frequencies above 26 Hz.



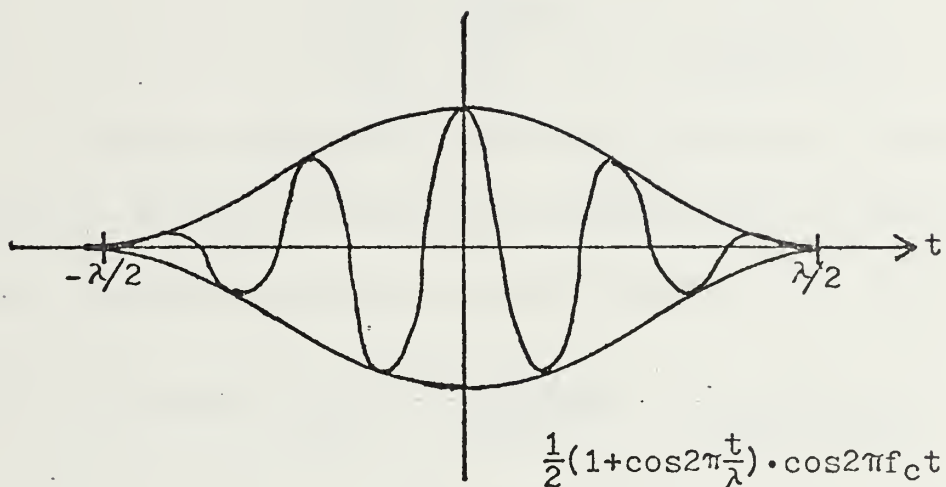


Figure 4-9. A single tegule enclosed in a cosine shaped envelope.



$$\begin{aligned}
F \text{ of envelope} &= \int_0^{\lambda/2} (1 + \cos 2\pi \frac{t}{\lambda}) \cos 2\pi f t \, dt \\
&= \frac{1}{2\pi f} \sin \pi f \lambda + \frac{1}{2} \left[ \frac{1}{2\pi (f - 1/\lambda)} \sin \pi (f - 1/\lambda) \lambda \right. \\
&\quad \left. - \frac{1}{2\pi (f + 1/\lambda)} \sin \pi (f + 1/\lambda) \lambda \right] \\
&= \frac{1}{2\pi} \sin \pi f \lambda \left[ \frac{1}{f} - \frac{1}{2(f - 1/\lambda)} - \frac{1}{2(f + 1/\lambda)} \right] \\
&= \frac{1}{2} \left( \frac{\sin \pi f \lambda}{\pi f \lambda} \right) \cdot \frac{1}{1/\lambda^2 - f^2}
\end{aligned}$$

This last expression is the Fourier transform of the envelope. The Fourier transform of the tegule enclosed in this envelope is simply this last expression convoluted with the transform of the tegule.

$$\begin{aligned}
F \text{ of tegule} &= \frac{1}{4\lambda} \left[ \frac{\sin \pi (f - f_c) \lambda}{\pi (f - f_c) \lambda} \cdot \frac{1}{1/\lambda^2 - (f - f_c)^2} \right. \\
&\quad \left. + \frac{\sin \pi (f + f_c) \lambda}{\pi (f + f_c) \lambda} \cdot \frac{1}{1/\lambda^2 - (f + f_c)^2} \right]
\end{aligned}$$

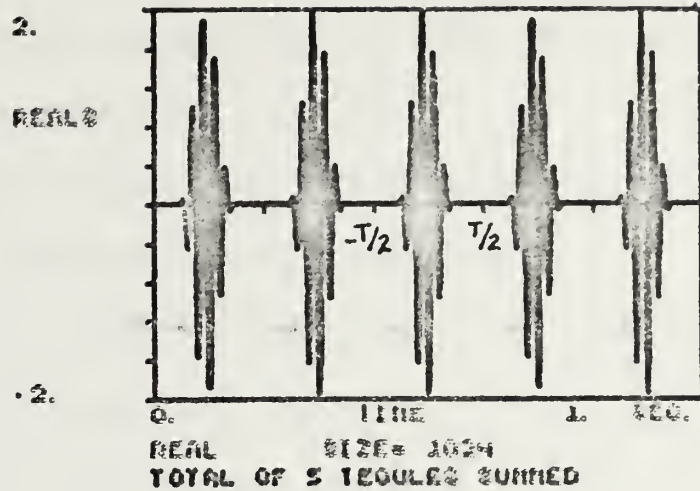
A plot of the Fourier transform of a single 50 Hz tegule with a cosine envelope of 100 ms duration is shown in dot representation in Figure 4-10b.

## 2. A Series of Tegules

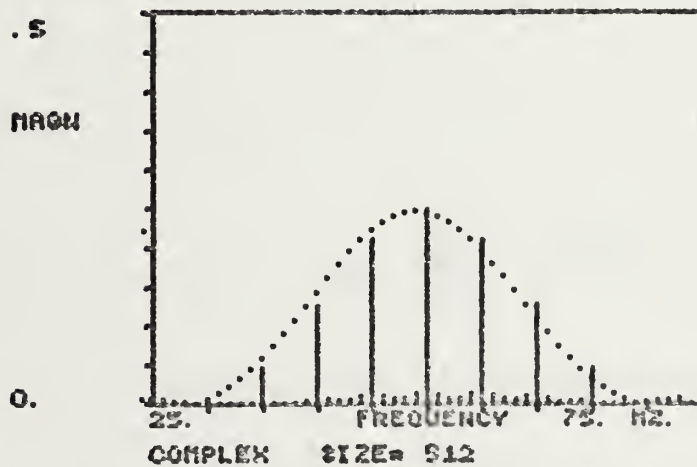
When a single tegule such as the one described above is repeated at regular intervals, the resulting Fourier transform has certain frequencies missing that would otherwise be present for a single tegule.







a. Series of 50 Hz tegules.



b. Frequency spectrum of a.

Figure 4-10. The frequency spectrum of a single 50 Hz cosine enveloped 100 ms tegule is shown in dot form in b. Bars in b represent the frequency spectrum of the series of 50 Hz tegules shown in a. Note how only certain discrete frequencies are left.



Figure 4-10a shows a series of tegules with the above characteristics. Each tegule has a frequency of 50 Hz and is enclosed in a cosine envelope with a duration of 100 ms. The tegules are evenly spaced with 100 ms separation.

The series of tegules may be represented by a summation of the mathematical representation for a single tegule.

Let

$x(t)$  = the original tegule

$y(t)$  = the summation of tegules

Then

$$y(t) = \sum_{k=-N}^N x(t-kT) \quad \text{where } N \text{ is the number}$$

of tegules and  $T$  is as shown in Figure 4-10a.

Let

$$F \left[ x(t) \right] = X(f)$$

Then

$$F \left[ y(t) \right] = Y(f) = X(f) \cdot \sum_{k=-N}^N e^{-j2\pi f k T}$$

To simplify the expression, one may use the identity

$$\sum_{k=-N}^N e^{-j\omega k T} = \left[ 1 + 2 \sum_{k=1}^N \cos \omega k T \right] \cdot \frac{\sin \omega T / 2}{\sin \omega T / 2}$$

Where  $\omega$  has been substituted for  $2\pi f$  and the additional unity term has been added for further simplification.



Let  $\omega T/2 = a$ . Then

$$(1 + 2 \sum_{k=1}^N \cos 2ka) \sin a = \sin a - (\sin 3a - \sin a) +$$

$$(\sin 5a - \sin 3a) - \dots$$

$$+ \sin(2N+1)a - \sin(2N-1)a$$

$$= \sin(2N+1)a$$

$$\text{So } Y(f) = X(f) \cdot \left[ \frac{\sin(2N+1) \pi f T}{\sin \pi f T} \right]$$

The Fourier transform described mathematically above for the series of tegules shown in Figure 4-10a is shown in the bar representation in Figure 4-10b.

The comb shape of the frequency spectrum of this ideal example of a series of tegules is often seen although to a lesser degree in EEG data. This is illustrated in the next section.



## V. RESULTS

The EEG modeling program has proven useful for the investigation of the effects of digital filtering on finite sinusoids. Models of the EEG are presented with consideration being given to the statistical summation of sinusoids within the cortex. The possibility of frequency beating as a model is presented. Modeling in the frequency domain is shown with discussion.

### A. EFFECTS OF NARROW BAND FILTERING

The shape of the envelope enclosing a tegule is characterized by a unique frequency spectrum. The spectrum of a square shaped envelope has a primary lobe about the frequency of the tegule and several additional lobes on either side of this lobe. Examples of this follow.

#### 1. Envelope Shape Deformation

The frequency spectrum of a sinusoid of constant frequency and finite duration is shown in Figure 5-1a. It has a frequency of 40 Hz and a duration of 50 ms. The "spreading" of the frequency spectrum is due totally to the finite length of the sinusoid and is of significant magnitude in this case.

The "bumps" in the frequency spectrum are referred to as lobes. These small lobes contain information of the shape and duration of the envelope of the sinusoid. Consequently, when a narrow band pass





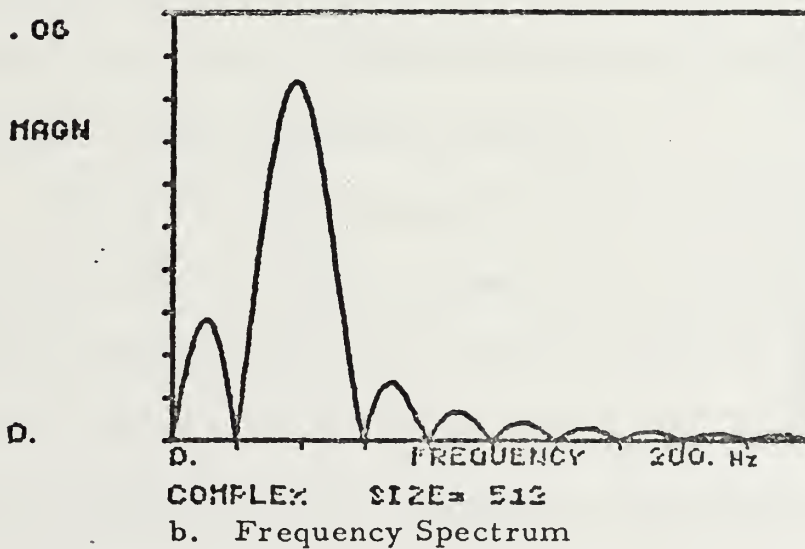
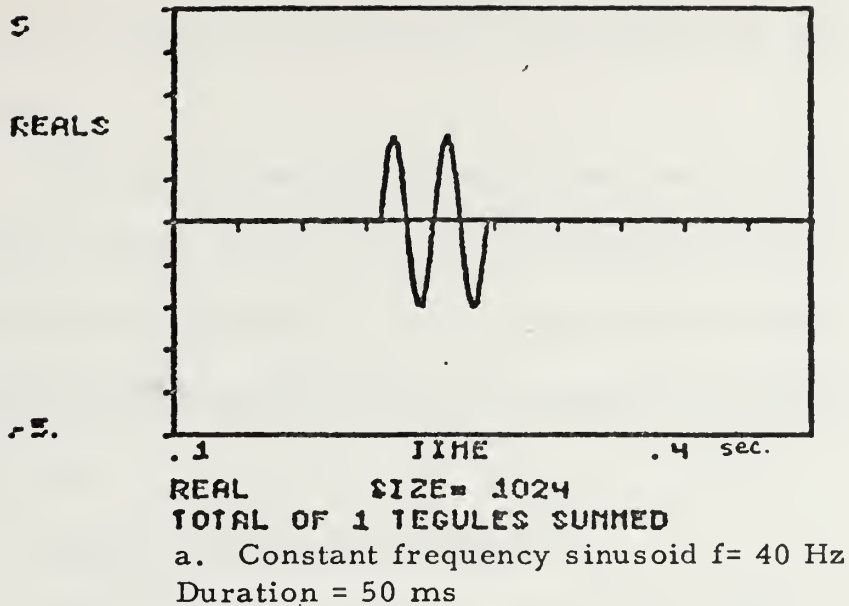


Figure 5-1. Constant frequency sinusoid (a), and its corresponding frequency spectrum (b).



filter is applied to a sinusoid, the shape of the envelope of the sinusoid due to the loss of this information in the exclusion of the small lobes by the filter.

Figure 5-2a shows the result of narrow band filtering on the sinusoid of Figure 5-1a. The frequency spectrum of the sinusoid is repeated with the portion of the spectrum within the filter band pass indicated by the dark area. Note the resulting cosine envelope in Figure 5-2a.

## 2. Tegule Length and Amplitude

The bandwidth of a filter has the effect of determining just how much information about the original sinusoid is passed along. The narrower the bandwidth, the less information that is passed along. As a consequence of this, the length and amplitude of the filtered sinusoid are a function of the bandwidth of the filter.

Figure 5-3a when compared to Figure 5-2a illustrates the effects of different size bandwidth on the same sinusoid (see Figure 5-1a). Again, the frequency spectrum is shown in Figure 5-3b with the bandpass of the filter shown in the dark area. Note that the amplitude of the tegule increases with the increasing bandwidth, whereas the duration of the tegule decreases.

## 3. Frequency Shifts Near Filter Limits

When the filter bandpass falls to one side of the main frequency of a sinusoid as illustrated in Figure 5-4b, the resulting tegule is not



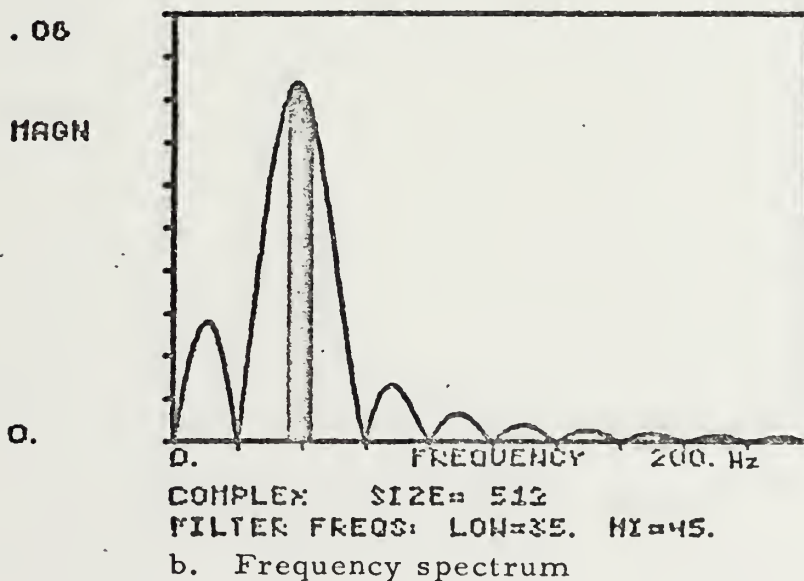
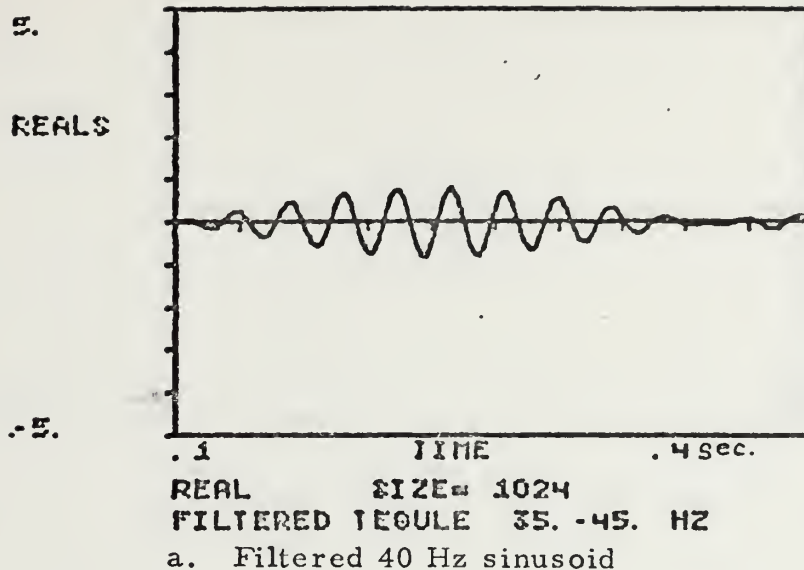


Figure 5-2. Result of narrow band filtering of the 40 Hz sinusoid of Figure 5-1a. Filter bandpass shown in dark in b.



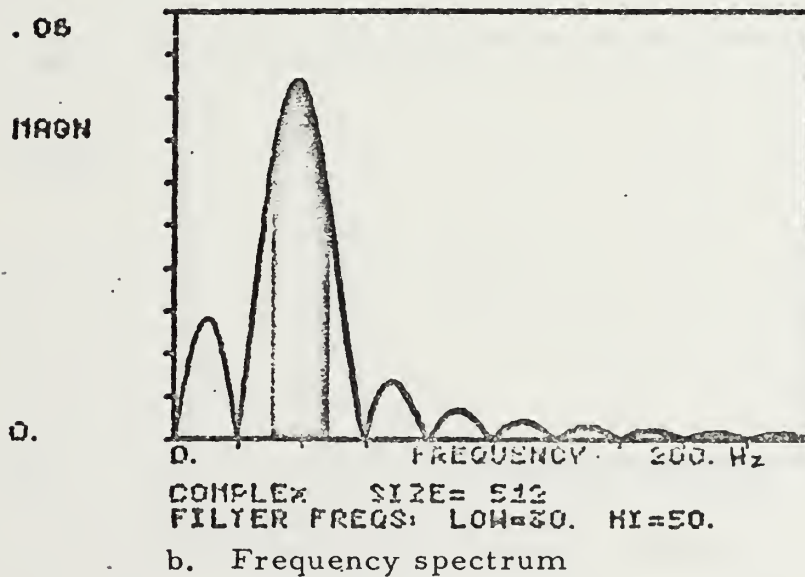
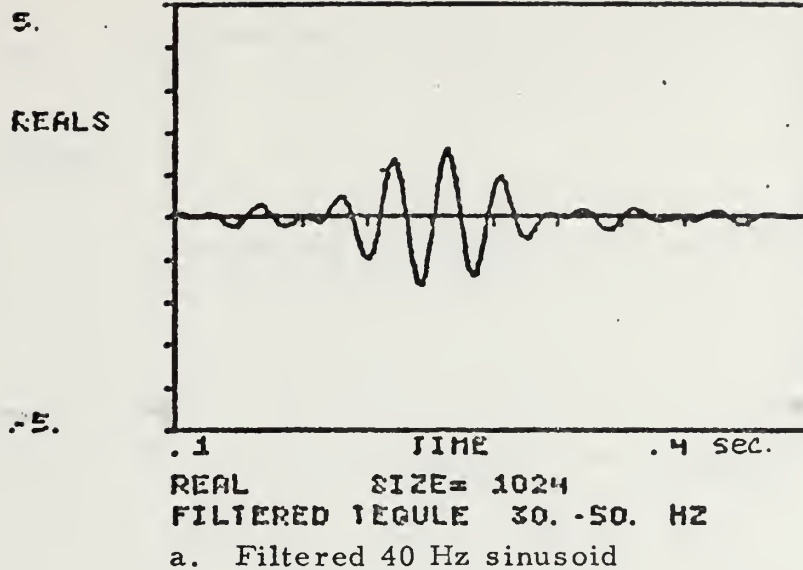


Figure 5-3. Result of a wider band filter.

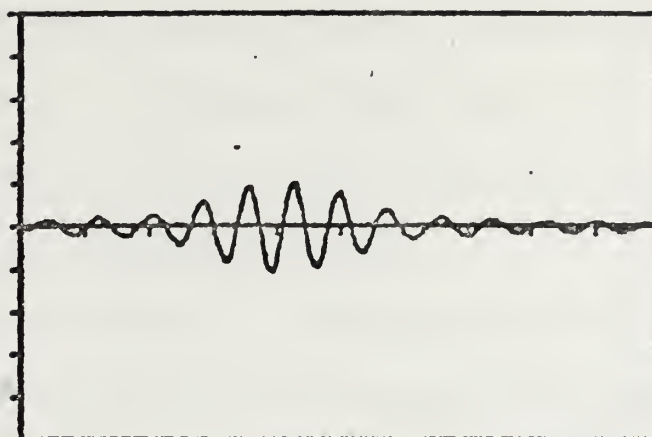




3.

REALS

.5.



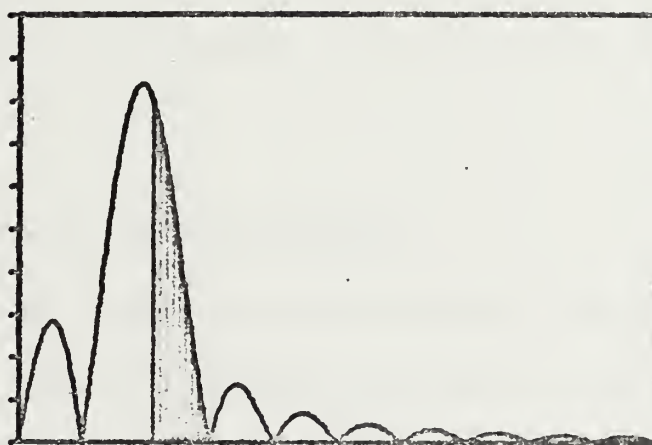
.1 TIME .4 sec.

REAL SIZE= 1024  
 FILTERED REGULE 40. -60. HZ  
 a. Filtered 40 Hz sinusoid

.06

MAGN

D.



0. FREQUENCY 200. Hz

COMPLEX SIZE= 512  
 FILTER FREQS: LOW=40. HI=60.  
 b. Frequency spectrum

Figure 5-4. Result of 40 Hz sinusoid being filtered with a band pass of 40-60 Hz.



only changed in envelope shape, amplitude and duration but also in frequency. The tegule resulting from the 40 to 60 Hz filtering of the sinusoid of Figure 5-1a is shown in Figure 5-4a. The frequency of this tegule is about 48 Hz as opposed to the 40 Hz of the original sinusoid.

#### 4. Frequency Changing Sinusoids

When the frequency of a sinusoid changes, as in the linear period sweep sinusoid of Figure 5-5a, the frequency spectrum contains more significant side lobes. Narrow band filtering of such a sinusoid has the effect of reducing the amount of frequency shift as well as altering all the other characteristics of the sinusoid as described in the above paragraphs. This is illustrated in Figure 5-6a. In this case, a 20Hz bandwidth filter was used as shown in Figure 5-6b. Nearly all of the frequency information of the sinusoid is contained within the main frequency lobe.

#### B. STATISTICAL SUMMATION MODEL

The theory that tegules may result from the addition of like sinusoids starting at times statistically distributed about a point in time is modeled using several types of sinusoids. It is postulated that the peak of the local alpha wave is the synchronizing mechanism for the start times of the sinusoids and that a 15 to 20 ms window exists during which all neural circuits contributint to the sinusoid will activate.



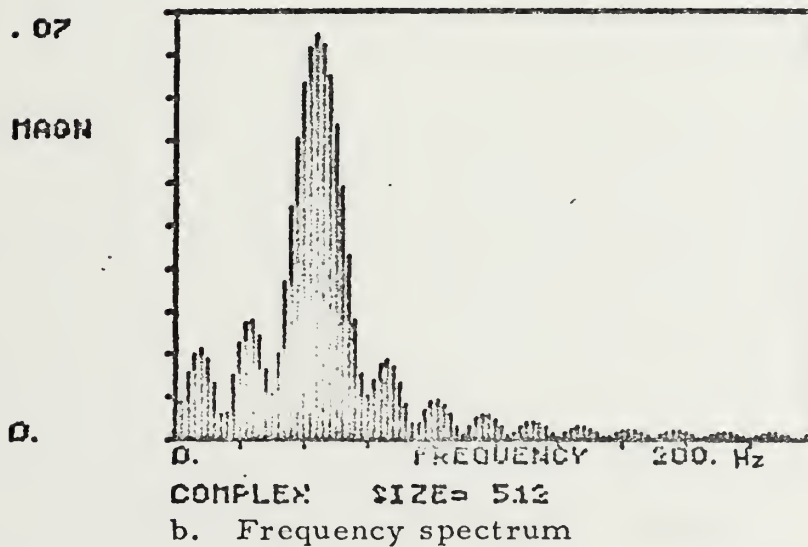
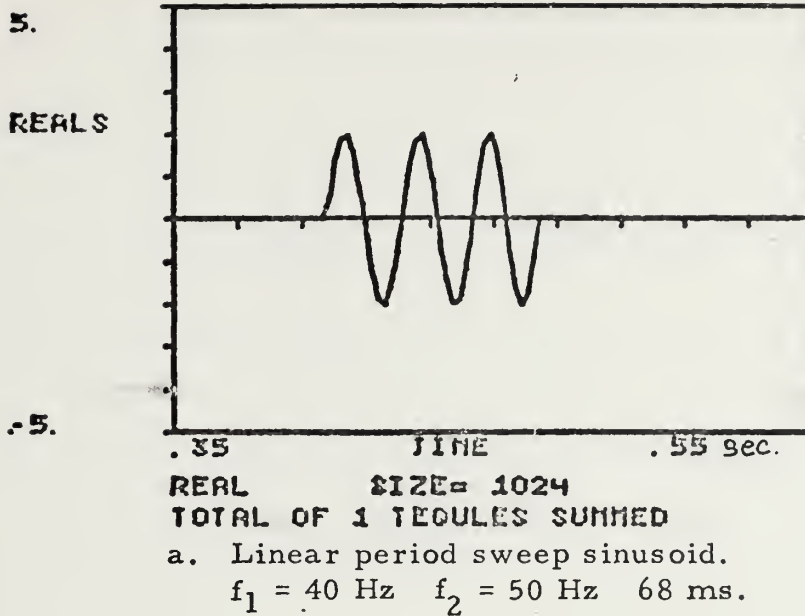


Figure 5-5. Linear period sweep sinusoid with frequency spectrum before filtering.



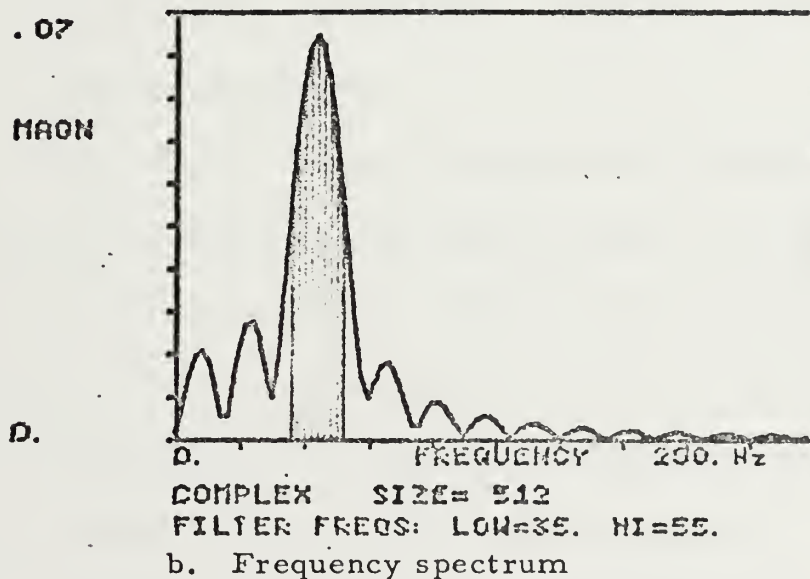
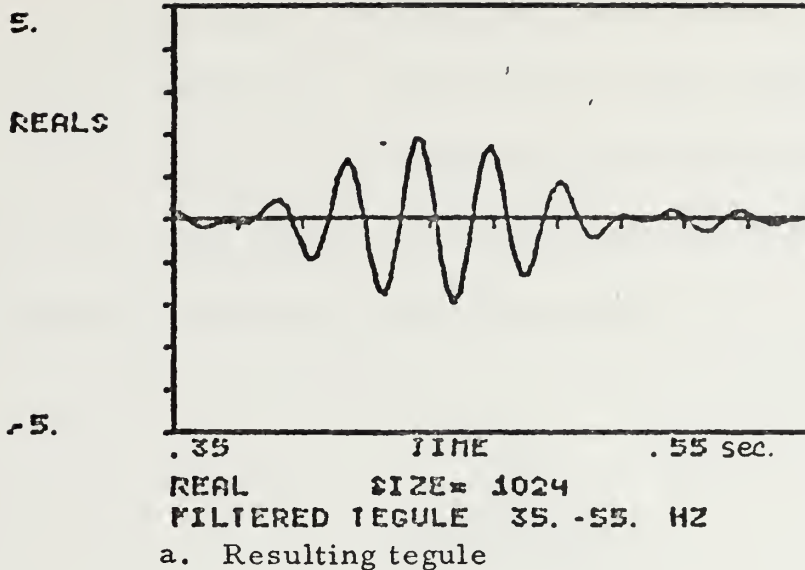


Figure 5-6. a. Result of filtering the linear period sweep sinusoid  
b. Filter band pass shown in dark.





## 1. Constant Frequency Model

A model using a constant frequency sinusoid of 40 Hz and duration of 50 ms is illustrated. Figure 5-7 is a plot of the discrete distribution of start times for the 19 identical sinusoids that were summed. Figure 5-8a is the original sinusoid generated and Figure 5-8b shows the composite sinusoid due to the summation.

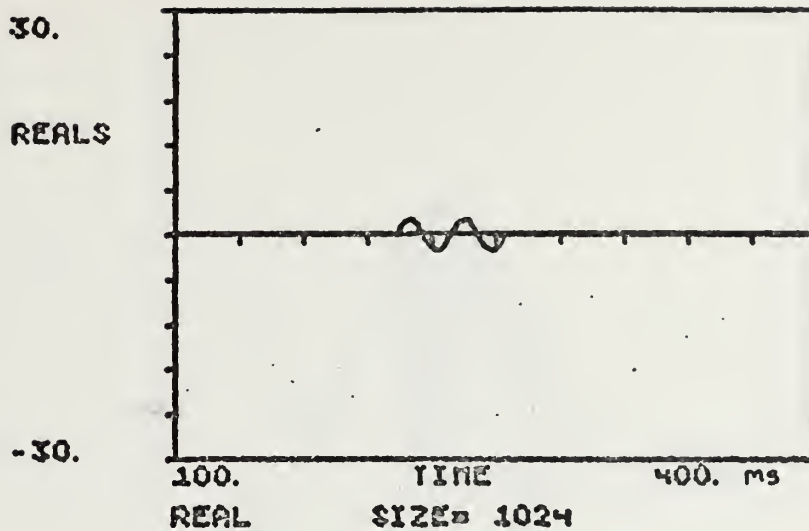
```
      XX
     XXX
    XXXXX
   XXXXXXXXXX
  XXXXXXXXXXXX
 200  208  216
```

Figure 5-7. Distribution of start times for sinusoids summed to generate the sinusoid on Figure 5-8b. Horizontal in milliseconds. Vertical represents number of sinusoids.

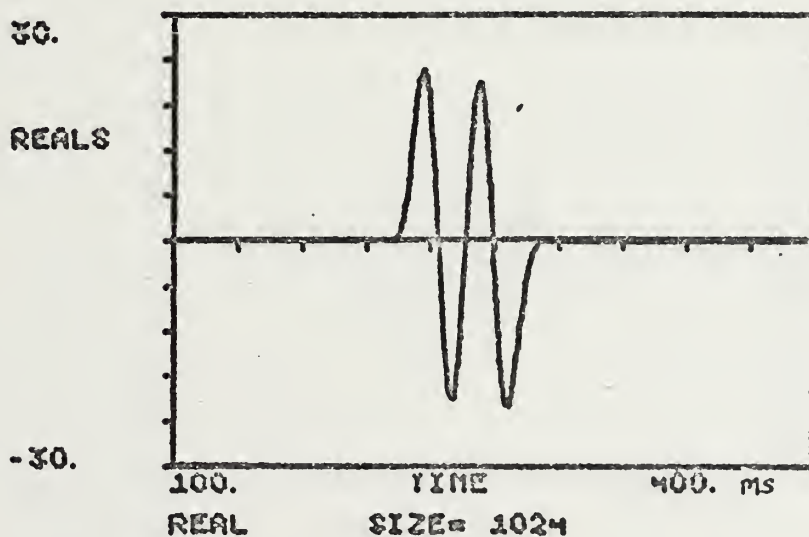
Each "X" in Figure 5-7 represents the addition of the original sinusoid to the composite sinusoid at the time over which it is placed. Note that an 18 ms period was chosen within which all the sinusoids were positioned. Since the digital program uses a discrete distribution whereas the brain would use a continuous distribution of start times, a certain degree of error is obvious in the modeling program.

Figure 5-9a shows the magnitude of the frequency spectrum of the composite sinusoid in the continuous line representation. The filter's bandpass is also shown and is represented by the vertical bars within the continuous line representation. The tegule that resulted from filtering





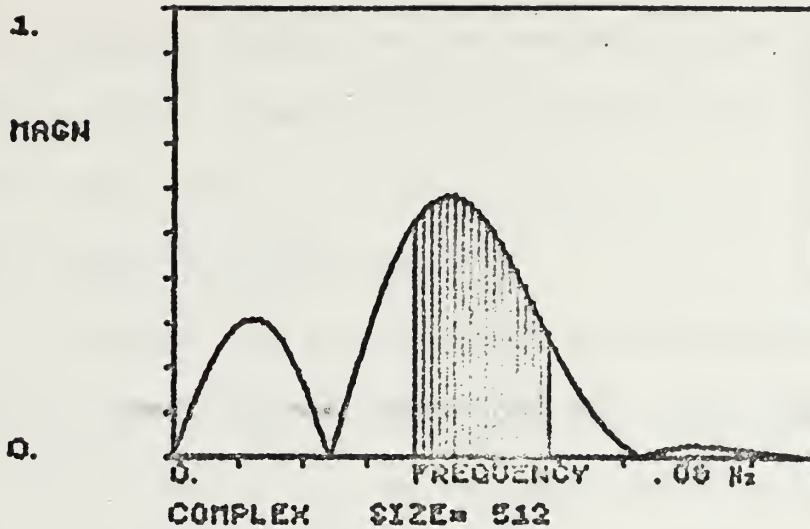
a. Constant frequency sinusoid.  
 $f = 40$  Hz.



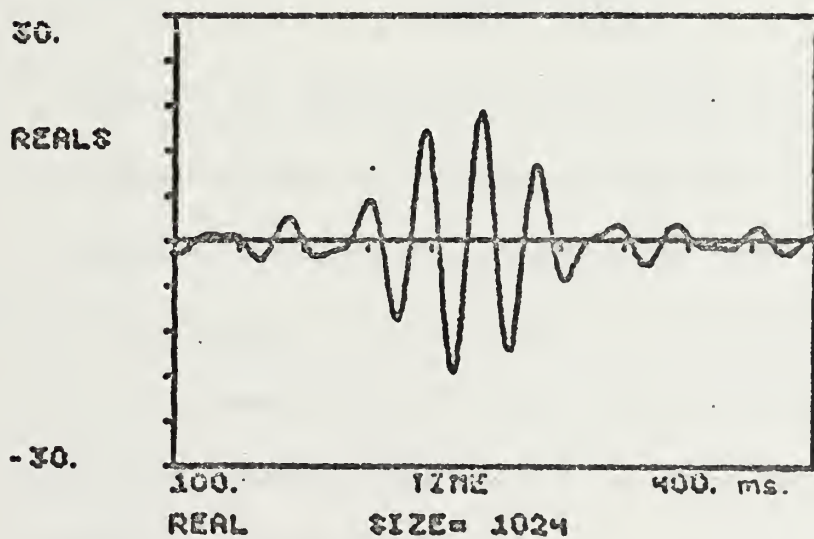
b. Composite of 19 sinusoids summed.

Figure 5-8. Constant frequency sinusoid. The sinusoid of a. was summed 19 times with a start time distribution as shown in Figure 5-7. The resulting composite sinusoid is shown in b. The tegule duration in a is 50 ms.





a. Filter frequencies: Low= 30 Hz  
Hi=50 Hz



b. Filtered tegule

Figure 5-9. a. Magnitude of FFT for sinusoid of Figure 5-8b. with filter bandpass shaded in. b. Tegule that resulted using a filter bandpass of 30 to 50 Hz.



with a bandpass of 30 to 50 Hz is shown in Figure 5-9b. Notice that the tegule is somewhat longer than the composite sinusoid but of the same frequency. Notice also that the envelope of the tegule is of the characteristic cosine shape.

## 2. Linear Period Sweep Model

A model using a linear period sweep between 40 and 50 Hz in a period of 110 ms was tested. A total of 21 identical sinusoids of the type shown in Figure 5-11a were summed with start times of each sinusoid as indicated by the distribution diagram in Figures 5-10 below.

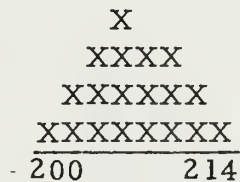


Figure 5-10. Distribution of start times for sinusoids summed to generate sinusoid in Figure 5-11b.

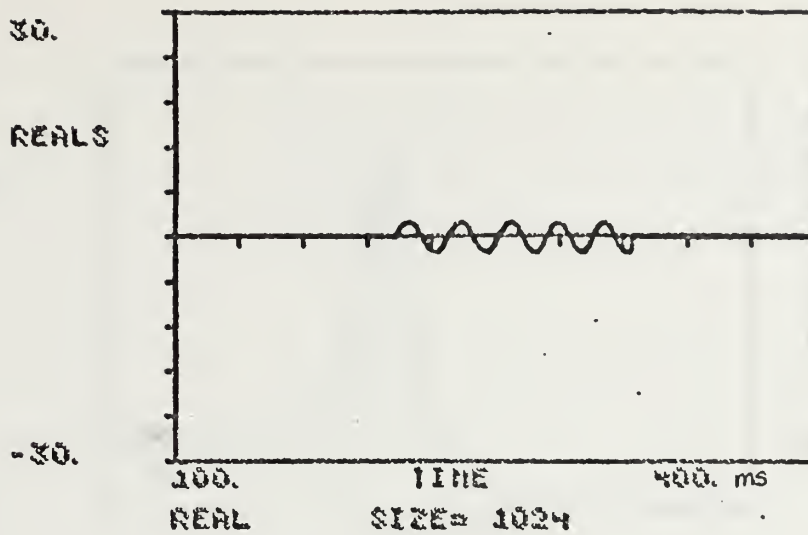
Figure 5-11b shows the composite sinusoid. Note the trapezoidal envelope shape. The magnitude of the FFT and the filtered FFT (dark area) are shown in Figure 5-12a. The filter limits used (35 to 55 Hz) produced the tegule in Figure 5-12b. Again, the characteristic cosine shaped envelope is noticeable.

## C. FREQUENCY BEATING MODEL

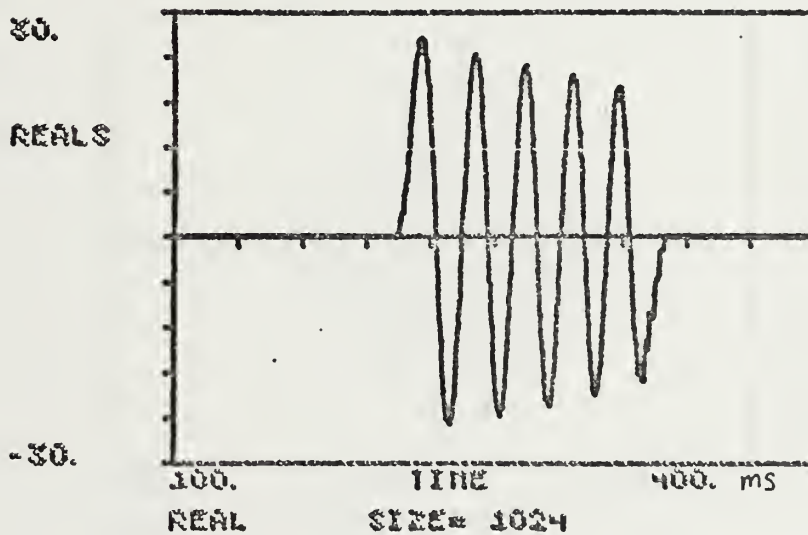
The mechanics of frequency beating were described in part 4-c of this paper. Figure 5-13a and b shows two sinusoids and the result of







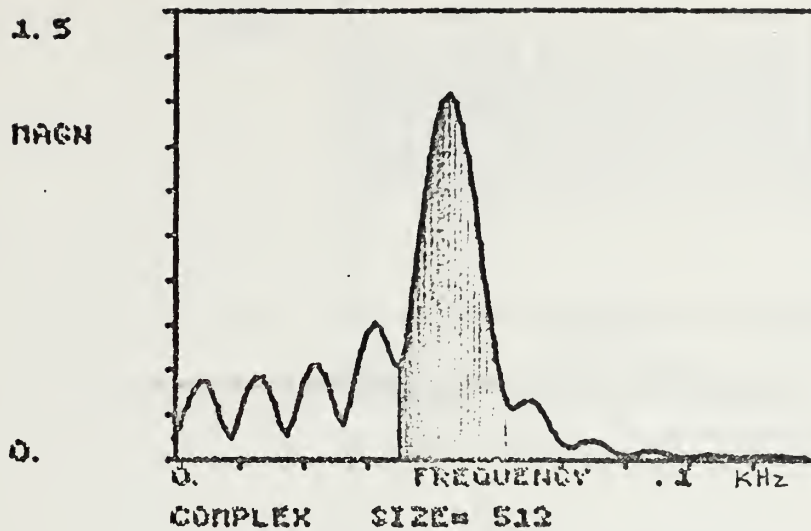
a. Original sinusoid



b. Composite of 21 sinusoids

Figure 5-11. A linear period sweep tegule. The sinusoid in a. was summed at times shown in Figure 5-10 to give the resulting sinusoid shown in b.





a. Magnitude of FFT of Figure 5-11b.  
Filter limits shown: Low=35 Hz H=55Hz

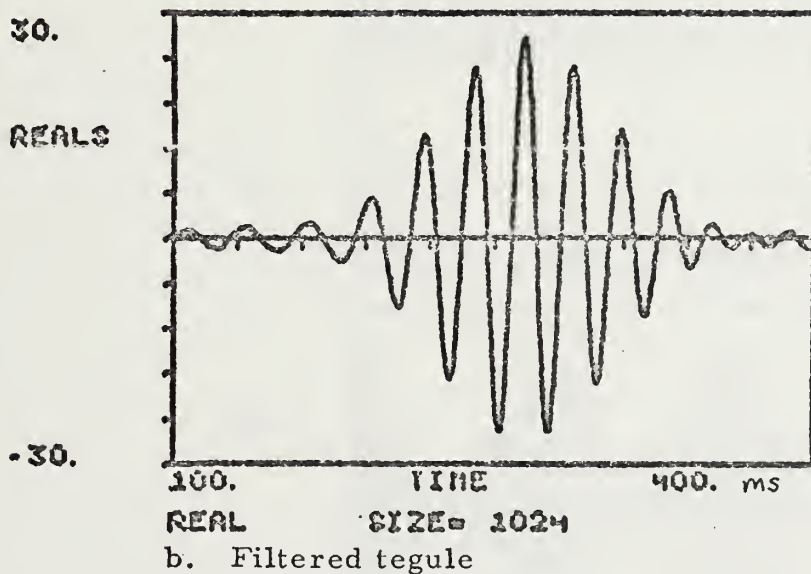
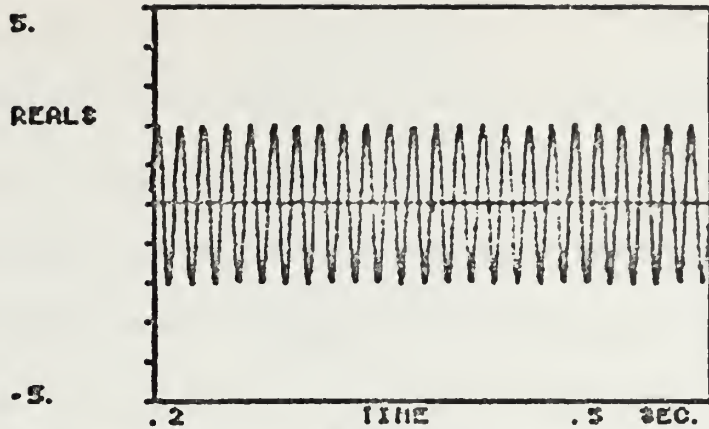
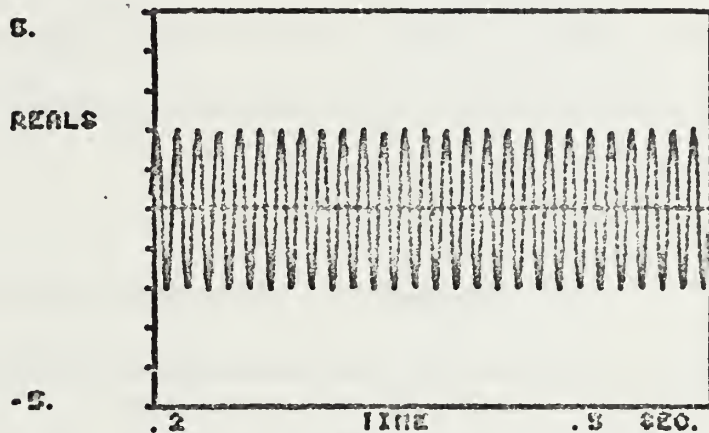


Figure 5-12. The resulting linear period sweep tegule after filtering with bandpass of 35 to 55 Hz.

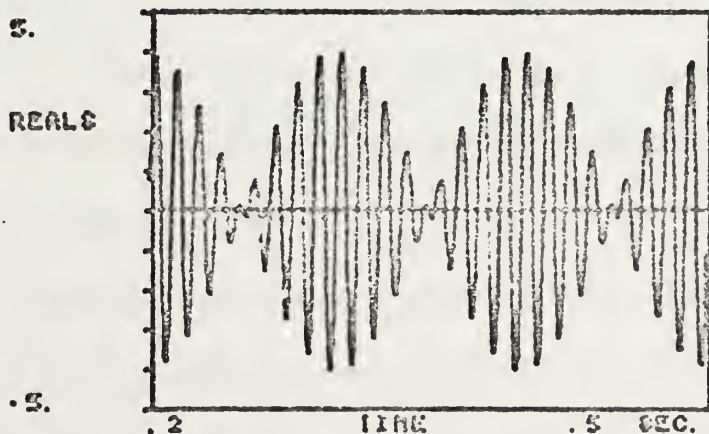




REAL SIZE= 1024  
a. TOTAL OF 1 TEGULES SUMMED



REAL SIZE= 1024  
b. TOTAL OF 1 TEGULES SUMMED



REAL SIZE= 1024  
c. TOTAL OF 2 TEGULES SUMMED

Figure 5-13. An 80 Hz and 90 Hz sinusoid are shown in a. and b. respectively. The result of linearly summing a. and b. is shown in c. Note the characteristic frequency beating shape.



linearly summing them to form the resulting series of tegules in 5-13c. The frequency spectrum of the tegules in 5-13c is simply two spikes, one at the frequency of each of the original sinusoids.

The frequency spectrum of two finite tegules of nearly the same frequency when summed linearly together with different start times is more like that observed in frequency spectrum of EEG data. Figure 4-15a through c show the summation of two such tegules. Figure 5-15a shows the frequency spectrum of these tegules. Figure 5-15b reveals that filtering with a 40 Hz filter reproduces the original tegule complex in its entirety.

#### D. MODELING IN THE FREQUENCY DOMAIN

Figure 5-16 shows the characteristic discreteness in the frequency spectrum of EEG data. This was discussed in Section IV of this thesis.

Figure 5-17a shows the modeled series of tegules that were used to produce the frequency spectrum in part b of the figure. Note how there are discrete frequencies surrounded by a frequency continuum and the similarity to those in Figure 5-16.

A total of six sinusoids with cosine shaped envelopes and four different frequencies were summed to create the wave form in Figure 15-17a. Notice that the 42.5 Hz tegule spectrum remained nearly continuous in shape with no discrete frequencies prevalent. This is due not only to the fact that the other tegules are relatively distant in frequency but also





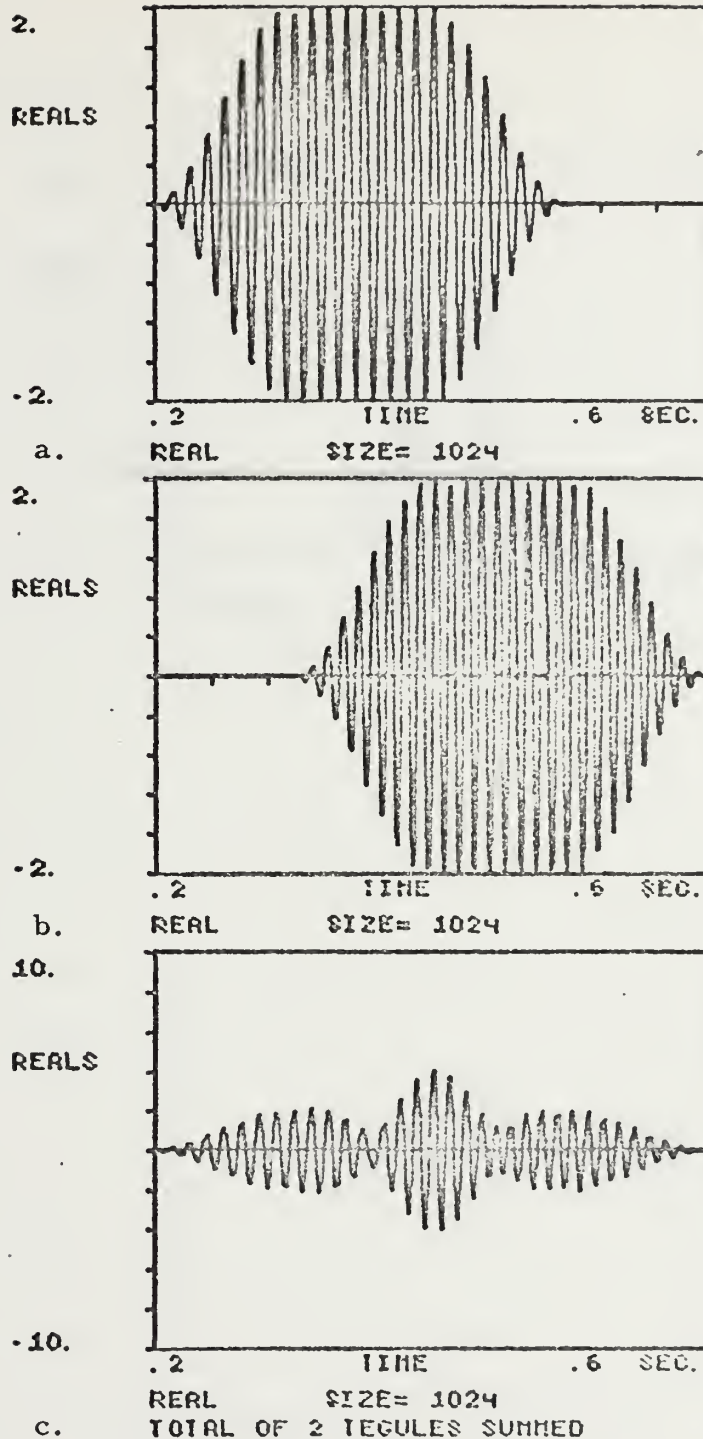


Figure 5-14. An 80 Hz and 90 Hz tegule with flat-topped cosine shape of 400ms duration are shown in a. and b. respectively. The result of their summation is shown in c.



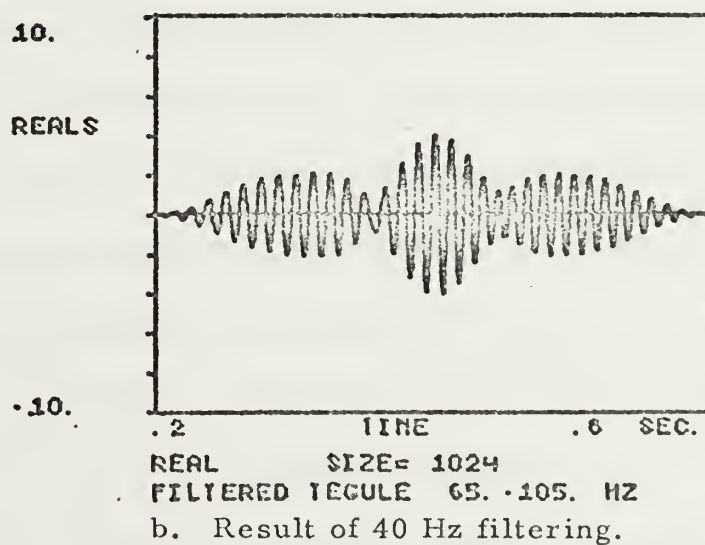
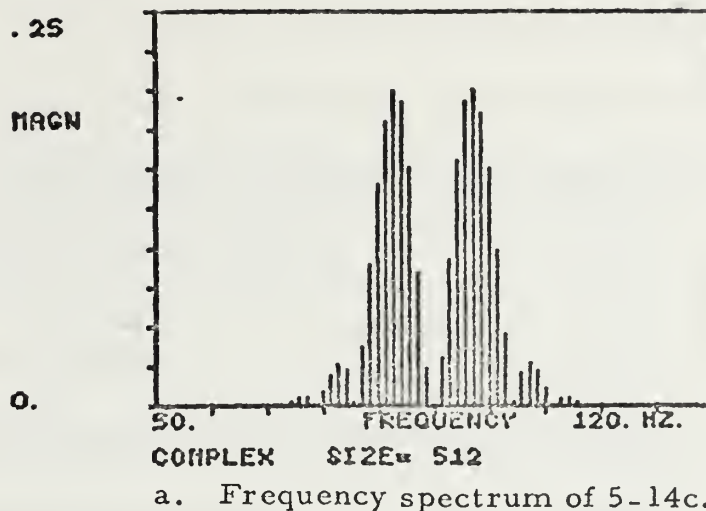


Figure 5-15. The frequency spectrum of the tegules in Figure 5-14c is shown in a. Note that a filter with a bandwidth of only 40 Hz reproduced the original tegules as shown in b. above.



since there is only one tegule of this frequency. This is illustrated by the 10 Hz tegules which are quite remote in the frequency domain from the other tegules. But since there are three tegules of 10 Hz, the ten Hz spectrum is missing certain frequencies that would otherwise have been present for a single tegule. This has caused what appears to be certain discrete frequencies to show up.

## E. THE ENVELOPE MODEL

The shape of the envelope of the sinusoid detected within the cortex is of prime interest in modeling the EEG. This is examined in the following subparagraphs.

### 1. A Comparison on a Single Tegule Level

It has been noted that the tegules that result from narrow band filtering have a cosine shaped envelope. This is true whether or not the original sinusoid of finite duration has a square or cosine shaped envelope. Figure 5-18 through 5-19 serve to illustrate this point.

The sinusoids in Figure 5-18 have identical characteristics except for their envelope shape. The difference in their frequency spectrums is quite pronounced. Note in Figure 5-17a that there are significant side lobes due to the frequency information required to describe a square shaped envelope. However, when a 20 Hz filter is applied, the remaining frequency information is nearly identical except for magnitude as shown in Figure 5-20. Consequently, the filtered tegules in Figure 5-21 differ only in amplitude.



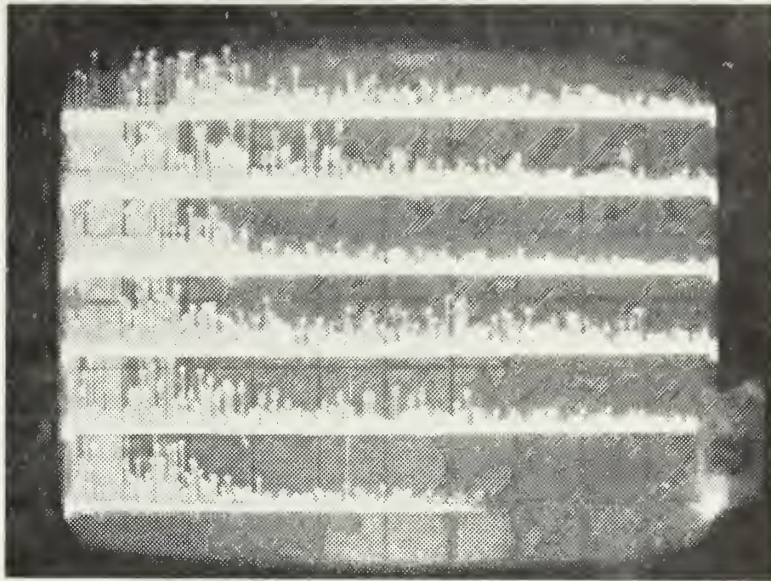
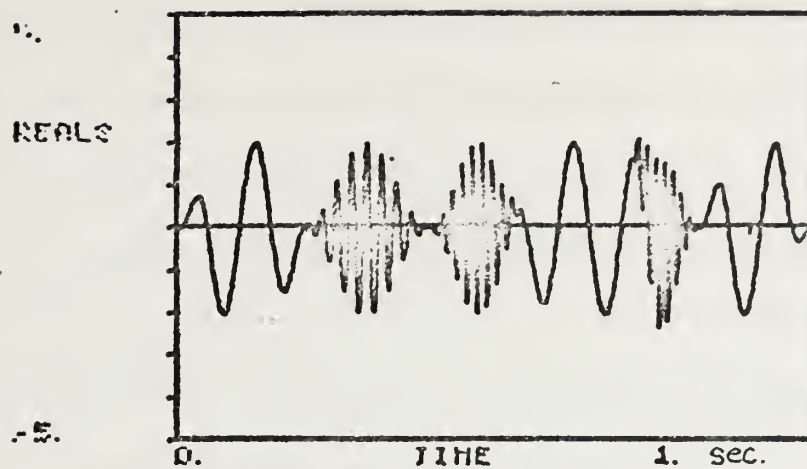


Figure 5-16. Frequency spectrum display of six channels of EEG data. The X-axis represents a range of 0 to 128 Hz. Note that there are discrete frequencies like those of Figure 5-15.

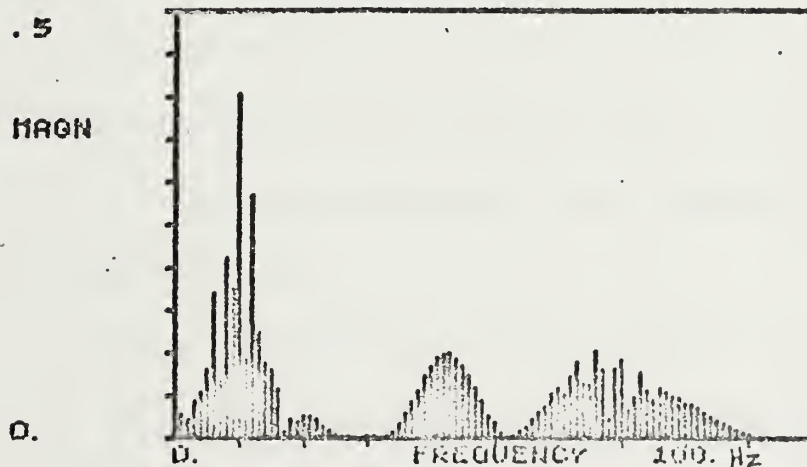






REAL SIZE= 1024

a. A series of tegules summed



COMPLEX SIZE= 512

b. Frequency spectrum of wave form.

Figure 5-17. Modeling in the frequency domain. Six tegules were summed to create a. The frequency spectrum resulting in b should be compared with Figure 5-16. Tegule frequencies are 10, 42.5, 64.5, and 74.3 Hz.



The above comparison of single tegules that differ only in envelope shape points to the fact that the detected tegules of EEG data using narrow bandpass digital filtering leaves uncertainty as to the actual envelope characteristic of the sinusoid within the cortex.

## 2. Square Vs. Cosine Envelope in the Frequency Domain

To determine the characteristic of the envelope of the sinusoid within the cortex, one is forced to examine the frequency domain in more detail. The first dissection of the EEG data occurs in the frequency domain presentation. In this form, one is able to observe the total make up of the EEG in a relatively clear presentation.

The nature of the frequency spectrum has a great deal of information concerning the shape of the envelope of the sinusoids that it represents. This was obvious in the case of the frequency spectrum of the single sinusoids as shown in Figure 5-19. In the case of several sinusoids, the frequency spectrum is more complicated but contains the information desired.

Figure 5-22a shows six identical 50 Hz cosine shaped enveloped sinusoids summed at quasi random times. The duration of each sinusoid is 100 ms. The frequency spectrum of the wave forms is shown in Figure 5-22b. Note how certain frequencies appear discretely. The otherwise smoothly shaped continuum that would exist for a single sinusoid of this type has been lost due to certain frequencies being reduced in magnitude.



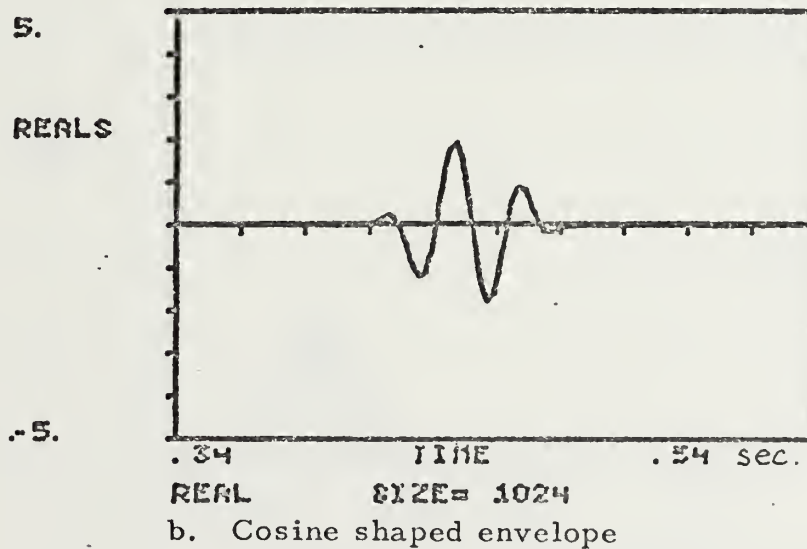
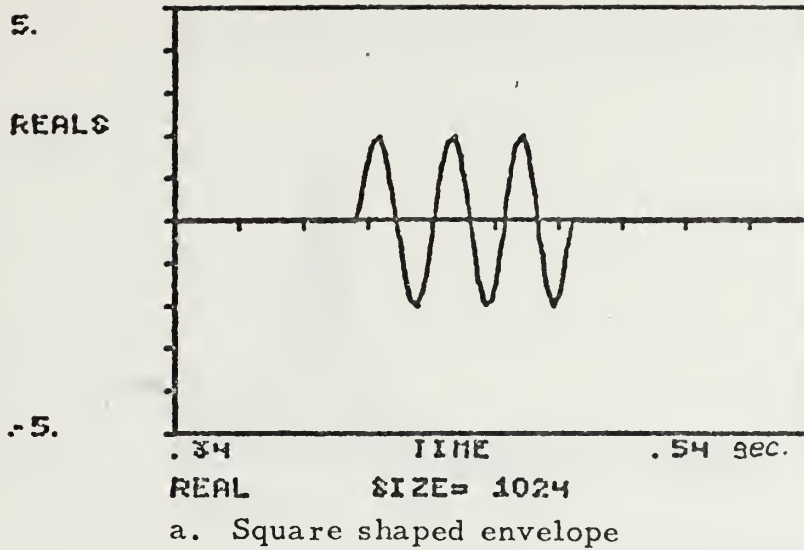


Figure 5-18. Two sinusoids that differ only in envelope shape. Both are linear period sweep sinusoids, 40 to 50 Hz in 68 ms.



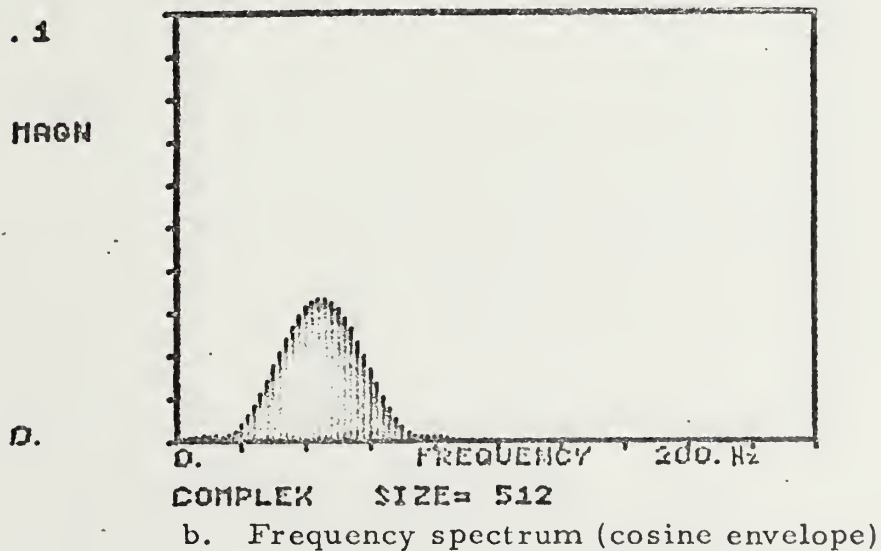
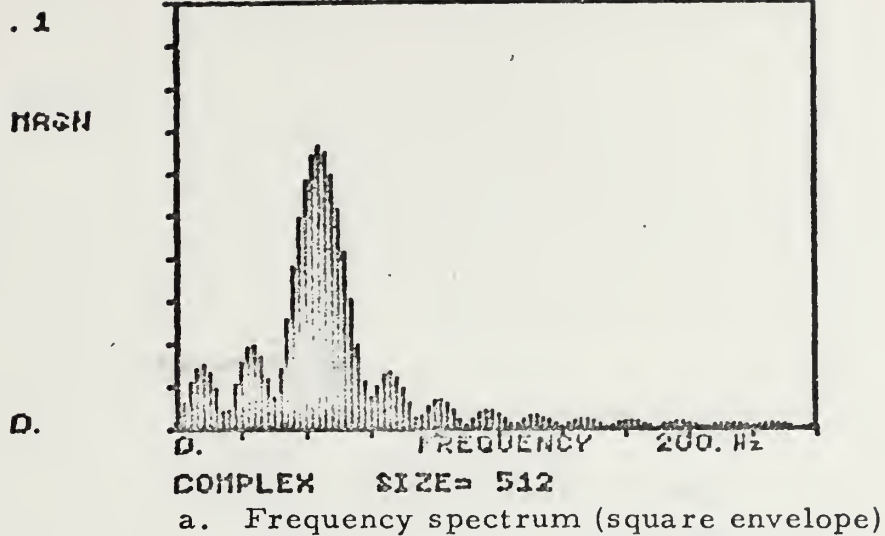


Figure 5-19. Frequency spectrums of sinusoids in Figure 5-18. Note the distinct differences between the spectrum of the sinusoid with the square shaped envelope in a and that of the cosine shaped envelope in b.

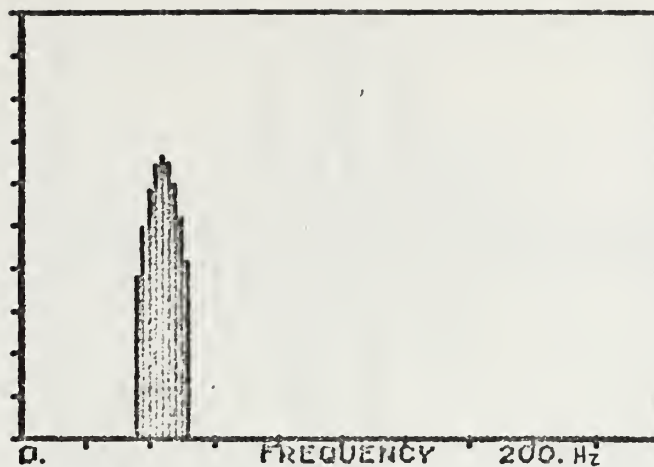




.1

MAGN

0.



COMPLEX SIZE= 512

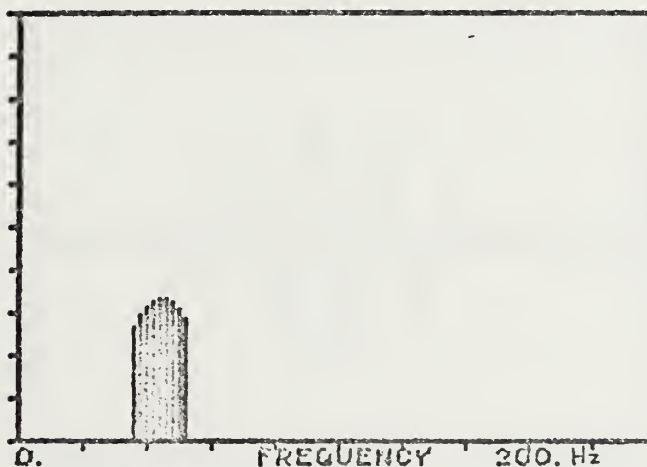
FILTER FREQS: LOW=35. HI=55.

a. Filtered frequency spectrum of square shaped envelope tegule.

.1

MAGN

0.



COMPLEX SIZE= 512

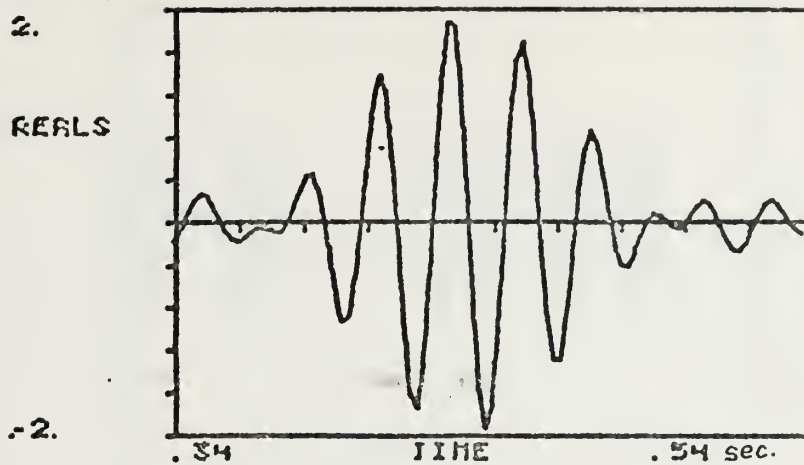
FILTER FREQS: LOW=35. HI=55.

b. Filtered frequency spectrum of cosine shaped envelope tegule.

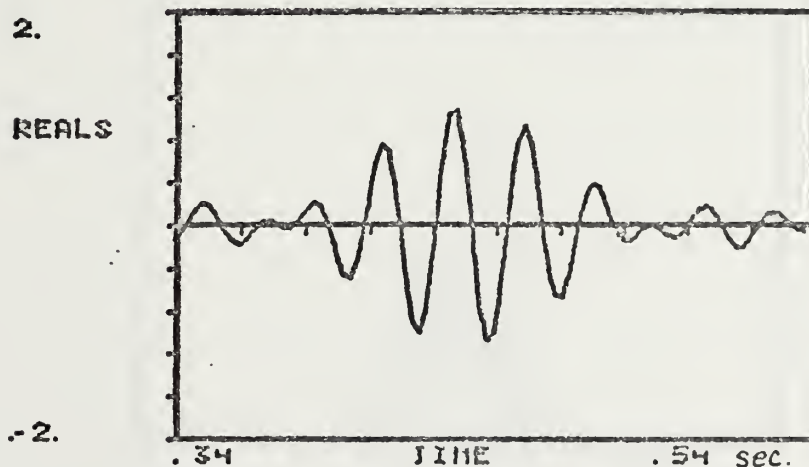
Figure 5-20. Filtered spectrums of sinusoids in Figure 5-18.

Note similarities in shape.





REAL SIZE= 1024  
 FILTERED TEGULE 35. -55. HZ  
 a. Square shaped envelope originally.



REAL SIZE= 1024  
 FILTERED TEGULE 35. -55. HZ  
 b. Cosine shaped envelope originally

Figure 5-21. The filtered tegules resulting from a bandpass of 35 to 55 Hz. Although the original sinusoids in Figure 5-18 differed in the shape of their envelopes, the tegules above differ only significantly in amplitude.



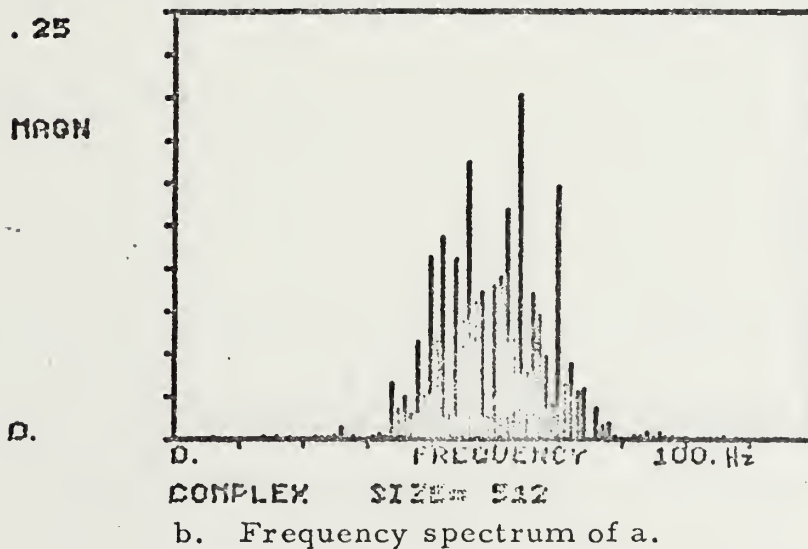
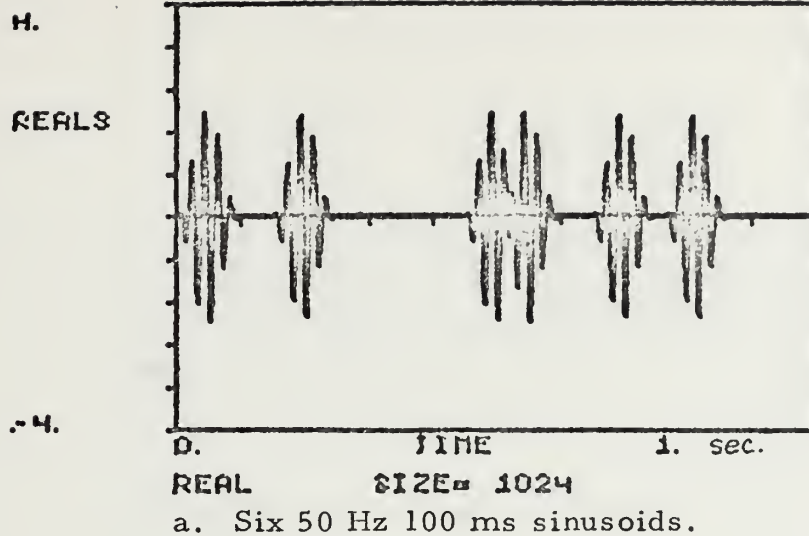


Figure 5-22. Six identical sinusoids with cosine shaped envelopes are added at quasi random times. The resulting frequency spectrum is shown in b.



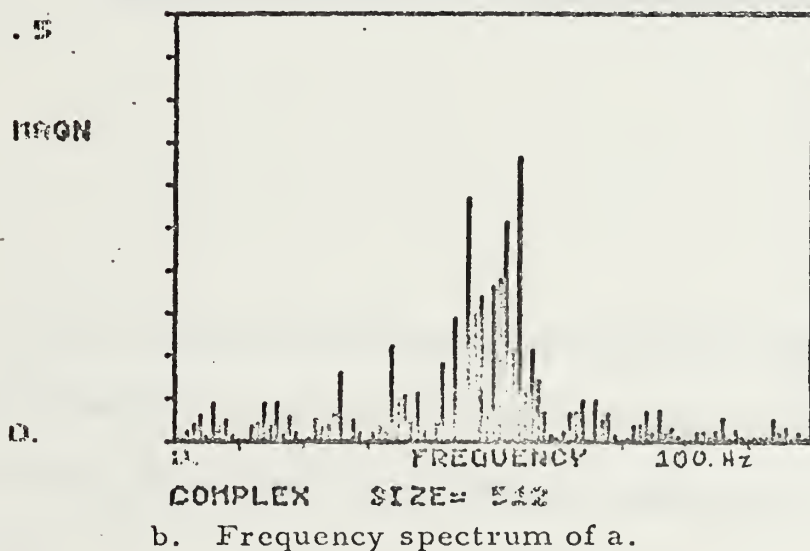
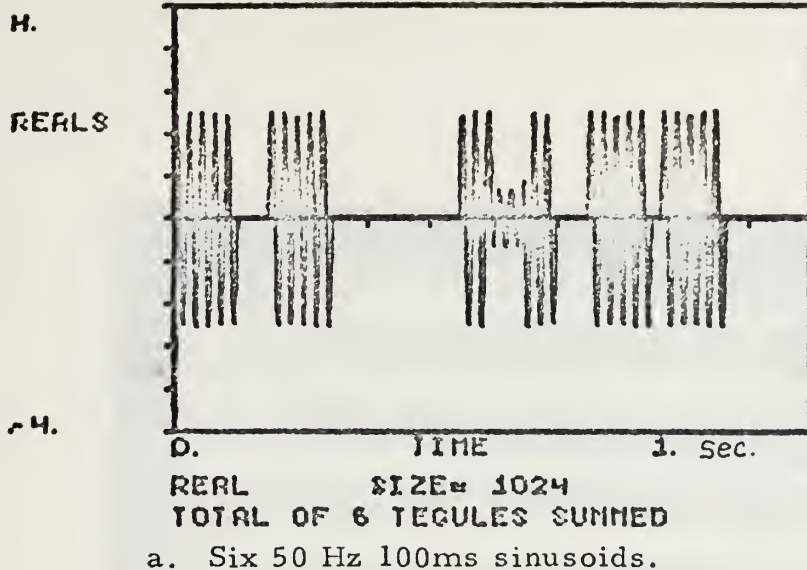


Figure 5-23. Square shaped envelopes used on sinusoids otherwise identical to those in Figure 5-22a. The resulting frequency spectrum is broader and more discrete in b than in Figure 5-22b.





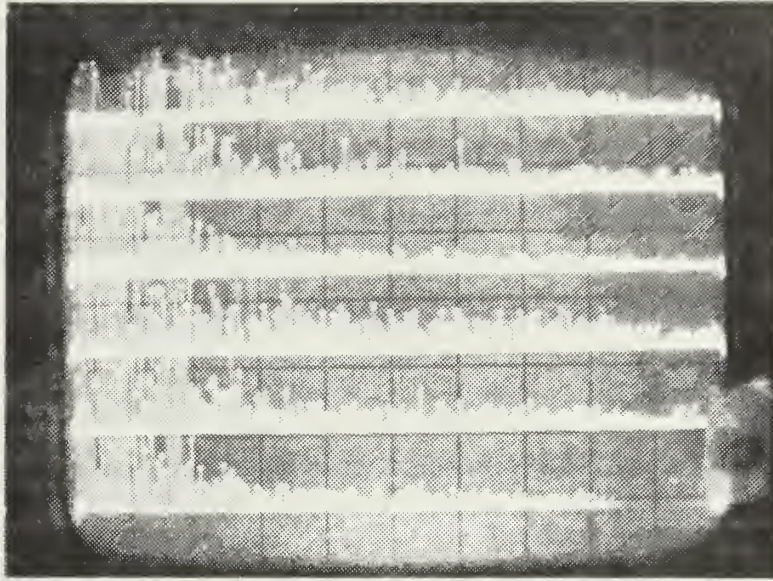


Figure 5-24. Frequency spectrums of six channels of EEG data. The X axis represents a range of 0 to 128 Hz. Frequencies below 26 Hz are beyond scale. Those above 26 Hz should be compared to Figures 5-22b and 5-23b.



In Figure 5-23a, an identical set of sinusoids was created differing from those in Figure 5-22a only in the square shaped envelope. The resulting spectrum in Figure 5-23b is even more discrete (notice vertical scale differences) than those in Figure 5-22b. Notice that the frequencies in 5-23b are spread out and are more similar to those in Figure 5-24 of EEG data than are the frequencies in Figure 5-22b. This would imply that the sinusoids within the cortex have envelopes that are more square in shape.

Figure 5-25 shows 250 ms of the tegules that resulted after filtering the sinusoids of Figure 5-23a with a 40-60 Hz bandpass. This is quite similar to the tegules in Figure 5-26 which are EEG data filtered with a 50-60 Hz bandpass. This is misleading, however, due to the narrow bandwidth of the filter used to produce the display in Figure 5-26.

Figure 5-27 is a display of EEG data filtered with a bandpass of 65-105 Hz. Note that in this data there appear cosine and flat-topped cosine enveloped tegules. An extensive review of EEG data that was filtered with a bandwidth of 40 or more Hertz revealed the same type of wave forms. No tegule had an exactly square envelope.

### 3. The Flat-Topped Cosine Envelope

The above discussion involved the two extreme envelope shapes of either square or cosine. A combination of these two shapes has been called the flat-topped cosine shape. Figure 5-28 a shows the result of summing nine flat-topped cosine enveloped tegules of various frequencies



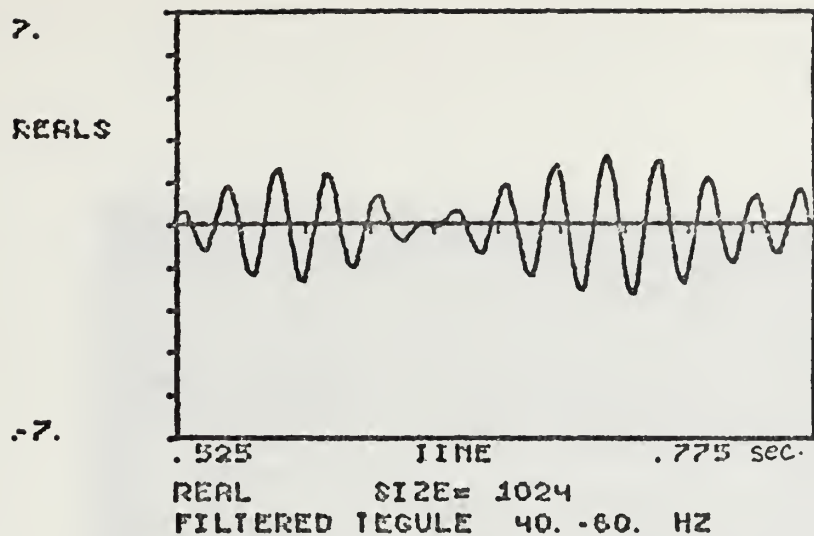


Figure 5-25. The sinusoids of Figure 5-23a were filtered with a band-pass of 40-60 Hz with the resulting tegules in this figure expanded to 250 ms size. Compare with Figure 5-26.



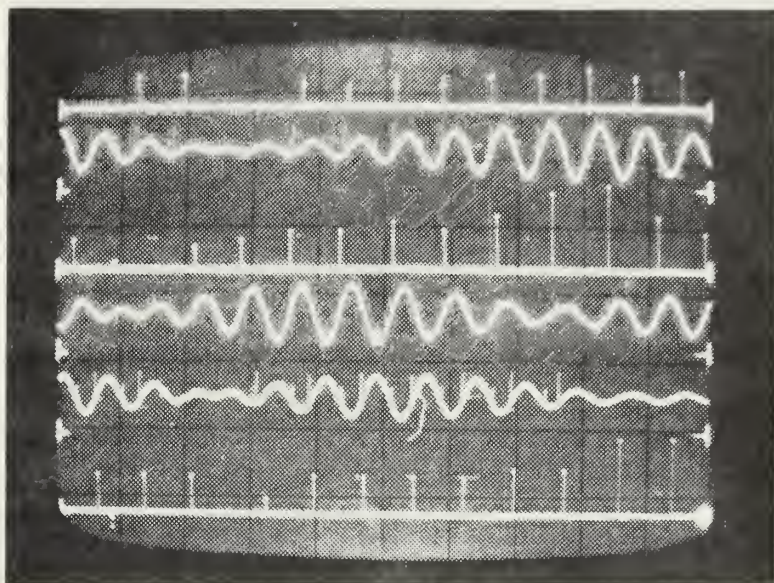


Figure 5-26. EEG data filtered with a bandpass of 50-60 Hz. Each trace represents succeeding 250 ms of data.





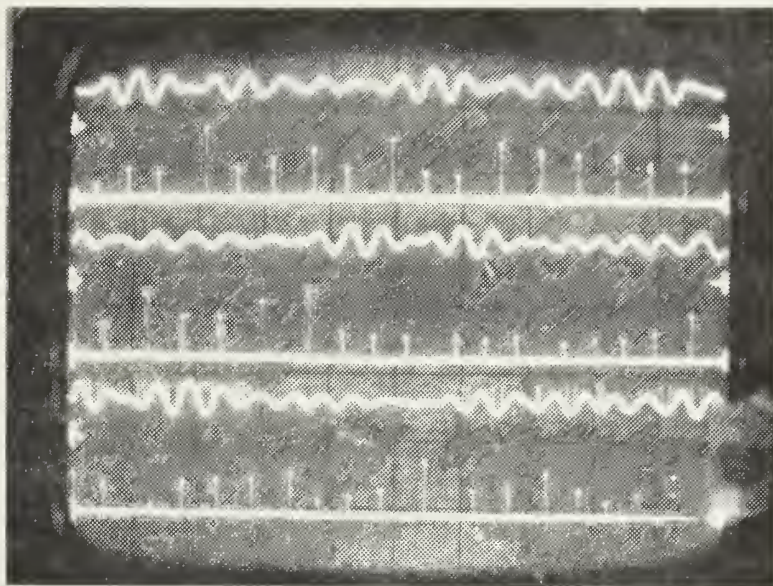


Figure 5-27. EEG data filtered with a bandpass of 65-105 Hz. Each trace represents 250 ms of consecutive data. Note flat-topped cosine envelope shapes.



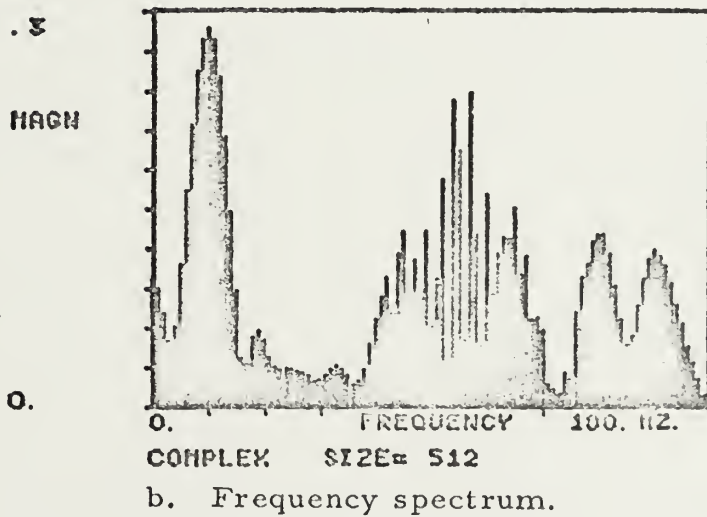
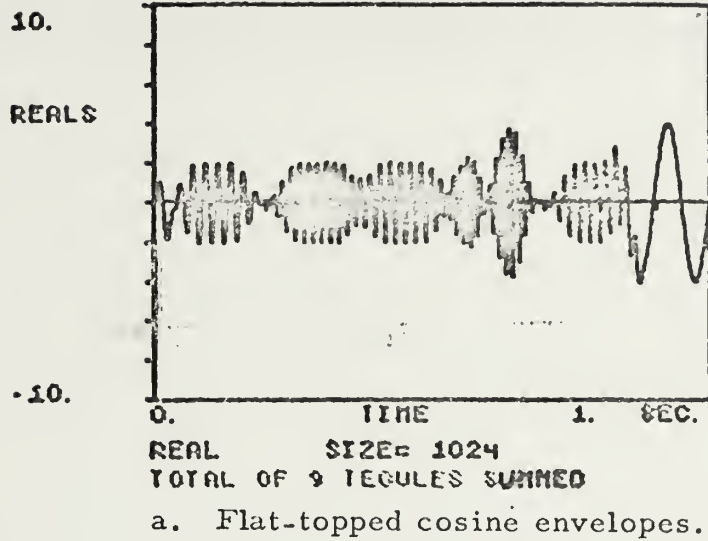


Figure 5-28. Nine flat-topped cosine enveloped tegules were summed to give result displayed in a. Tegules were as follows: 80 Hz, 200 ms at .5 sec; 90 Hz, 150 ms at .55 sec; 64.5 Hz, 200 ms at .2 sec; 45 Hz, 180 ms at .025 sec; 55 Hz, 200 ms at .7 and .35 sec; 25 Hz, 50 ms at 0 sec; 2-10 Hz, 300 ms at .8 sec.



and durations. Two tegules were summed around .55 second to produce the effect of frequency beating as well. The frequency spectrum of this series of tegules is displayed in b. of the figure and is similar to that of EEG data.



## VI. DISCUSSION AND CONCLUSIONS

This section concerns the results of modeling as it applies to various areas of the research program. Reduction and loss of information is discussed. A possible source of tegules and their envelope shape are described and areas of further research are indicated. Pattern recognition using modeling as a confirmation is discussed.

### A. LOSS OF INFORMATION

A general truism is that information processing never increases information content. This is also true of EEG research. But the problem in EEG research is not one of too little information but rather one of far too much. Consequently, the problem becomes one of selecting the correct amount and type of information to obtain meaningful results.

#### 1. Filtering

The existence of frequencies higher than 50 Hz in the EEG has been confirmed through the use of digital filtering in the region of 65-105 Hz. This is information that has always been available but masked by the much stronger lower frequencies that predominate in the EEG. How these higher frequencies tie in with mental processes has yet to be established. There are indications of "preferred frequencies" in this region of the frequency spectrum. Confirmation of this may follow shortly with further research in this area.





## 2. Limiting Information

The key to pattern recognition in the EEG may lie in the area of selective information presentation to the person whose EEG is being processed. The ability of the human brain to subconsciously locate a particular nerve ending by audio and visual feedback to the subject of the impulses to the particular nerve ending has fostered much encouragement in this area. It is postulated that the human brain may be able to communicate with itself at the subconscious level if proper presentation of the EEG is fed back. Such a scheme might provide some interesting results. This would necessarily involve limiting the information contained in the EEG just as is the case in the single nerve ending phenomena and may be developed in the not too distant future.

### B. SOURCE OF TEGULES

The question as to what exactly is the source of the tegules observed in the digitally filtered EEG has not been fully answered. Research in this area involves the coordinated effort of microelectrode and EEG techniques. Work done in this area has resulted in a strong belief that the reverberating circuit is the answer [Ref. 1].

The proposition that the alpha wave has a strobing effect on reverberating circuits has also been all but confirmed [Ref. 2]. The decrease in alpha with a corresponding increase in higher frequency activity during thought processes has been termed alpha blocking and by its very nature, strongly implies the interdependence of higher frequency activity with the alpha.



## C. DISCRETE FREQUENCIES

Research during the past year has strongly indicated that there are preferred frequencies in the EEG above 20 Hz. The frequency spectrum of the EEG often will show certain discrete frequencies that exist for several consecutive seconds of data.

### 1. Series of Tegules

Modeling has shown that a series of sinusoids in an epoch of data causes the frequency spectrum to exhibit the discrete frequency appearance that is so prevalent in EEG data. This is most predominant in all the EEG data obtained and is indicative that tegules exist within the cortex at discrete frequencies.

### 2. Indications of Patterns

The fact that certain discrete frequencies often are present during consecutive epochs of EEG data is strongly indicative of the repetition of a particular pattern in the EEG. The discrete frequencies observed are not necessarily the same as those of the original sinusoids. In fact, modeling indicates that they most likely are not. Further research in this area is needed.

## D. ENVELOPE SHAPE

The answer to the question as to the shape of the envelope of the tegules within the cortex seems to have been answered by filtering with a 40 Hz or wider bandwidth. Modeling has indicated that the discrete appearance of the frequency spectrum of the EEG is caused more by a series of tegules than the shape of their individual envelopes.



The stochastic properties of nerve axons is more in support of the cosine shaped envelope. But if the alpha wave form is in fact a strobing pulse every 100 ms, one would expect that most axons involved in tegules that last for less than 100 ms would be triggered within approximately 20 ms of each other. Such a concept has been modeled and the resulting wave form envelopes have been more square than not.

The cause of the flat-topped cosine shape of the envelope, that is most prevalent in wide band EEG data, may be caused by the facilitation and inhibition of stellate cells synapsing with the apical dendrites or cell body level dendrites of pyramidal cells. This could cause the initial number of pyramidal cells involved with the reverberating circuit to increase suddenly. In a like manner, inhibition or fatigue could cause the final number of pyramidal cells to drop suddenly. Such a process would cause the tegules to exhibit the flat-topped cosine shape prevalent in EEG data.

#### E. FURTHER RESEARCH

The computer modeling program used in this thesis has great potential for further study in many areas of EEG research. Research in the area of pattern recognition in the frequency domain will need substantial confirmation through modeling techniques.

The computer modeling program also has contributed significantly in determining what kind of information is lost using digital filtering.



This has already prompted the creation of new techniques in processing the EEG. The use of the averaging electrode referencing has proven to reduce the amount of information that is common throughout the cortex. This enables one to concentrate on the activity in a particular area of the cortex while averaging out the activity common throughout the cortex.





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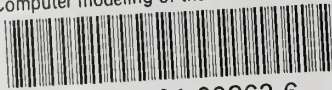
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